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MULTIFRACTAL ANALYSIS OF DAILY US COVID-19 CASES

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***Abstract*—In this work, we applied the multifractal detrended fluctuation analysis (MFDA) to analyze the highly irregular behavior or volatility clustering of daily COVID-19 cases in the United States. We use the multifractal spectrum of the MFDFA to characterize the path and predict short or long-memory behavior of the time series on different time scales. Empirical results from the generalized Hurst exponent (gHE) and multifractal spectrum estimation indicates that path of the COVID-19 cases is multifractal and keeps becoming less fractal as the days progresses.**

***Index Terms*—Multifractal detrended fluctuation analysis, multifractal spectrum, volatility clustering, long-memory behavior, time series, COVID-19.**

I. Introduction

The COVID-19 pandemic caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), a respiratory illness has claimed millions of lives. The COVID-19 and associated economic shutdown created economic crises around the world. In fact, the World Health Organization (WHO) categorized it as a public health emergency and since then, several precautionary measures have been put in place to protect lives (e.g. mask mandate, social distancing) as well as policies to recover the economy (e.g. stimulus payments). In order to explain the dynamics of the virus and find effective approaches to model the spread, several researchers are actively working on this topic. The goal of this study is to analyze and predict the behavior of daily US COVID-19 cases in the near future using multifractal detrended fluctuation analysis (MFDFA).

II. Data

The time series used in this paper are collected daily through an ongoing GitHub repository of data on coronavirus cases and deaths in the US by the New York Times (NYT) starting January 21, 2020 with 422 observations. Due to extreme fluctuations in the time series, we before we start our analysis we transform the data using the relation:

$$D_t = log(C_t) - log(C_{t-1}) \tag{1}$$

where D_t denotes the log-difference of daily cumulative cases COVID-19 C_t on day t. Figure 1 displays the time evolution of the Covid-19 cases (Upper panel) and the log-difference Covid-19 cases (Lower panel).

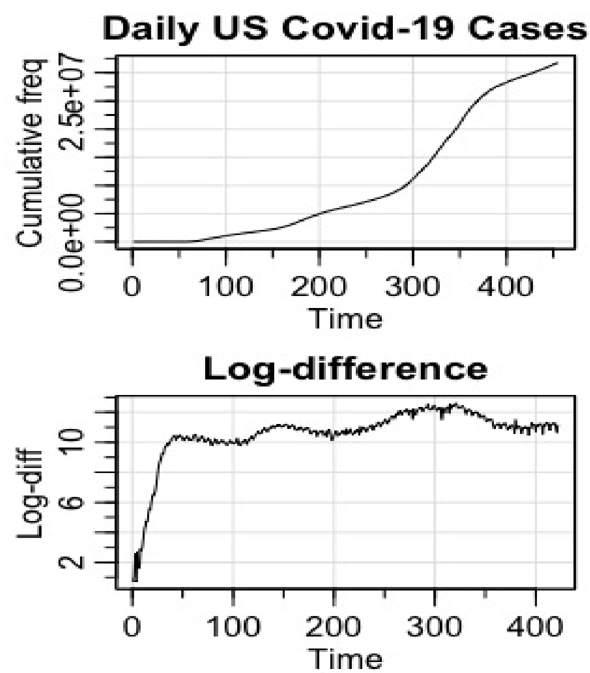

Fig. 1: Evolution of daily US cumulative and log-difference of covid-19 cases

III. Multifractal Detrended Fluctuation Analysis (MFDFA)

The profile of the cumulative series X_t is divided into M_s segments of equal length s as detailed by Kantelhardt in [1]. To ensure that all data points are included, the same number of segments M_s going the opposite direction is constructed giving $2M_s$ segments. Least-square regression is then applied to estimate the local trends in each segment of length u from equation (2) below. Next, the average segments where a qth order fluctuation $\mathbb{F}_q$ is collected as shown in equation (3):

$$\mathbb{F}_S^2(u) = 1/s \sum_{i=1}^{s} \left[X_u(i) - \bar{X_u}(i)\right]^2 \tag{2}$$

$$\mathbb{F}_q(S) = \left\{ 1/2M_s \sum_{u=1}^{2M_s} \left[\mathbb{F}_S^2(u)\right]^{q/2} \right\}^{1/q} \tag{3}$$

In order to generate the scaling behavior of fluctuations from the slope $H(q)$ of log-log plots of $\mathbb{F}_q^2(S)$ for each q versus segment length s [2] we use the relation:

$$\mathbb{F}_q(S) \propto S^{H(q)} \implies \log\left[\mathbb{F}_q(S)\right] - H(q)\log(S) = 0 \tag{4}$$

From the generalized Hurst exponent $H(q)$, we formulate the mass exponent in equation (5), singularity exponent in equation (6) and singularity spectrum in equation (7) given below:

$$\tau(q) = qH(q) - 1 \tag{5}$$

$$\alpha = H(q) + qH'(q) \tag{6}$$

$$f(\alpha) = q\left[\alpha - H(q)\right] + 1 \tag{7}$$

We remark that at $q = 2$, the MFDFA algorithm is the same as monofractal DFA. Long-memory is depicted by the scaling exponents for instances where $H(q) \in [0.5, 1]$ and $H(q) \in (0, 0.5)$ indicates short-memory behavior.

IV. Empirical Results

We set the length of segments as $26 < s < 54$ with increments of $\Delta s = 5$ to perform the MFDFA. We also set the $q - order$ weights of the local fluctuations to lie within the interval $[-6, 6]$.

TABLE I: MFDFA Exponents

$q - order$	$\tau(q)$	$H(q)$	α	$f(\alpha)$
-6	-6.4218	0.9036	0.9447	0.7536
-5	-5.4840	0.8968	0.9338	0.8149
-4	-4.5576	0.8894	0.9195	0.8795
-3	-3.6456	0.8819	0.9004	0.9443
-2	-2.7513	0.8757	0.8769	0.9976
-1	-1.8751	0.8751	0.8628	1.0122
0	-1.0000	0.8873	0.8873	1.0000
1	-0.0971	0.9029	0.8900	0.9870
2	0.7799	0.8900	0.8055	0.8311
3	1.5433	0.8478	0.6983	0.5516
4	2.1917	0.7979	0.6250	0.3081
5	2.7735	0.7547	0.5857	0.1550
6	3.3254	0.7209	-	-

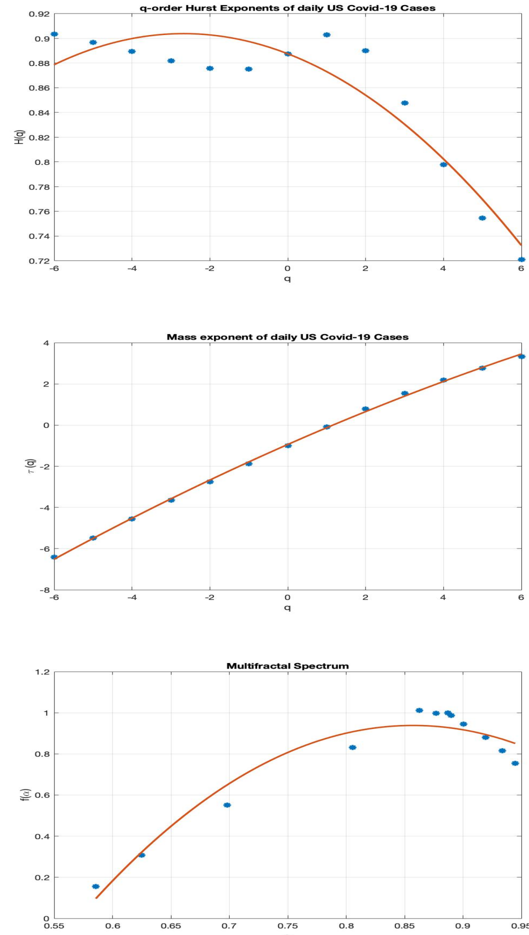

Fig. 2: Plots of q-order Hurst exponents, mass exponents and singularity spectrum respectively.

V. Discussion

Table I and Figure 2 (Upper panel) shows an estimated $H(q) = 0.89$ at $q = 2$ which denotes a long-memory behavior in the time series' path. However, the moderate variation in $H(q)$ are inversely related to changes in q over time. This indicates that, the coronavirus cases is getting better not worse.

The scaling function $\tau(q)$ of the multifractal process plots (Middle panel) is almost linear since the first data point was collected on Jan 21, 2020. This implies that the number of covid-19 cases become less fractal as time passes. A concave mass exponent plot indicates otherwise. Thus, the daily cases are steady.

The spectrum (Lower panel) peaks around $\alpha = 0.87$ and lies within a wide spectrum that ranges from 0.58 to 0.94 which suggests that volatility clustering in the series underlined by negative q tends to show long-memory behavior and vice versa. In contrast, the multifractal spectrum tends to have a long left tail because the series is characterized by a multifractal structure that is sensitive to high magnitudes of local fluctuations.

VI. CONCLUSION

In this paper, we detected the existence of long-memory behavior in the daily cases of Covid-19 in the United States using the generalized Hurst exponent (gHE) through the multifractal detrended fluctuation method. From the empirical results, the daily COVID-19 cases becomes less fractal even though the magnitude of change is steady. We also observe a high sensitivity to local fluctuations of the multifractal structure of the time series which suggests that the evolution of the COVID-19 cases will reduce drastically if we stay safe in our smaller communities (e.g. households) in the United States. The MFDA technique discussed in this study can be applied to other extreme events arising in geophysics; modeling the evolution of the magnitudes before and after a major earthquake and in finance; describing the behavior of a stock price before and after a financial crash.

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