2012 Hawaii University International Conferences
Education, Math & Engineering Technology
July 31st to August 2nd
Waikiki Beach Marriot Resort & Spa
Honolulu, Hawaii

Optimizing the Partnership between Academia and Industry: The Capstone Design Projects

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POINT-PUSHING PSEUDO-ANOSOV MAPS AND FILLING SIMPLE CLOSED GEODESICS ON RIEMANN SURFACES WITH PUNCTURES

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Let S be an analytically finite Riemann surface with type (p, n), where p is the genus and n is the number of punctures of S. Assume that $3p - 3 + n > 0$. Let $f : S \to S$ be a pseudo-Anosov map and let $a \subset S$ be a simple closed geodesic. Denote by $f^m(a)$ the geodesic homotopic to the image curve of a under the map f^m. It is well-known [3] that $\mathscr{S} = \{f^m(a) : \ m \in \mathbf{Z}\}$ fills S in the sense that the complement $S \backslash \mathscr{S}$ consists of polygons or once punctured polygons. Later, Fathi [2] showed that a finite subset of $\mathscr{S}$ fills S. It is natural to ask if any pair of elements of $\mathscr{S}$ also fills S. Unfortunately, the answer to this question is "no". In fact, From Theorem 1.1 of [6], for any two non-separating non-isotopic simple closed geodesics a, b on S, there is a pseudo-Anosov map f such that $f(a) = b$.

By contrast, in [4], Masur–Minsky showed that there is an integer K_0, independent of the choice of a, such that $(a, f^m(a))$ fills S for all integers $m \geq K_0$. To determine the smallest possible integer K with this property, Farb–Leininger–Margalit [1] considered the curve complex which is defined as follows. Let $\mathcal{C}_k$ denote the collection of all k-th dimensional simplexes. In particular, $\mathcal{C}_0$ is the set of vertices that can be identified with the set of homotopy classes of simple closed geodesics on S. Let $\mathcal{C}(S)$ be the collection of $\mathcal{C}_k$ for all $k \geq 0$. We then turn $\mathcal{C}(S)$ into a metric space by specifying that each edge is of length one, and define the path distance $d_{\mathcal{C}}(c, c')$, $c, c' \in \mathcal{C}_0$, by taking shortest paths along edges connecting c and c'. Let $\mu = \lim_{m\to\infty} \inf d_{\mathcal{C}}(a, f^m(a))/m$. Then by Proposition 3.6 of [4], $\mu > 0$ and μ is independent of the choice of a. It was shown in [1] that if m is the smallest integer so that $m\mu > 2$, then $(a, f^m(a))$ fills S.

In this presentation, we study the similar problem on a surface S that contains at least one puncture x. Write $\tilde{S} = S \cup \{x\}$. Let $\mathscr{F}$ be the set of pseudo-Anosov maps of S that are isotopic to the identity on $\tilde{S}$. Kra [5] proved that $\mathscr{F}$ is non-empty and contains infinitely many elements. Let $f \in \mathscr{F}$ and let $F : [0, 1] \times \tilde{S} \to \tilde{S}$ denote the isotopy between f and the identity as x is filled in. Then $\tilde{c} = F(t, x)$, $t \in [0, 1]$, is an oriented filling closed curve passing

Date: July 31, 2012.
1991 *Mathematics Subject Classification.* Primary 32G15; Secondary 30C60, 30F60.
Key words and phrases. Riemann surfaces, pseudo-Anosov, curve complex, filling curves.

through x in the sense that $\tilde{c}$ intersects every simple closed geodesic. Note that the curve $\tilde{c}$ is defined on $\tilde{S}$, not on S.

For an $a \in \mathcal{C}_0$, we denote by $\tilde{a}$ the simple closed geodesic on $\tilde{S}$ homotopic to a when a is viewed as a curve on $\tilde{S}$. Let $K = K(f)$ be the smallest integer such that $(a, f^m(a))$ fills S whenever $m \geq K$.

Theorem 1. [7] *For an element $f \in \mathscr{F}$ and $a \in \mathcal{C}_0$, we have $K \leq 3$ and the inequality is sharp. Furthermore, if $\tilde{a}$ is non-trivial and intersects $\tilde{c}$ more than once, then $K \leq 2$. If $\tilde{a}$ is trivial, then $K = 1$.*

Theorem 2. [7] *Let $f \in \mathscr{F}$ and let $a \in \mathcal{C}_0$. Assume that $(a, f^2(a))$ does not fill S. Then there is a unique simple closed geodesic b that is disjoint from both a and $f^2(a)$. Furthermore, $b = f(a)$ and $\{a, b\}$ forms the boundary of an x-punctured cylinder on S.*

An immediate consequence of Theorem 2 is the following corollary.

Corollary 1. [7] *Let $f \in \mathscr{F}$ and let $a \in \mathcal{C}_0$. Assume that a and $f(a)$ are not disjoint. Then $(a, f^m(a))$ for any $m \geq 2$ fills S.*

Our last result extends Theorem 1 and Theorem 2, which gives concrete examples for vetices in $\mathcal{C}_0$ such that their distances are greater than three.

Theorem 3. [8] *For any $f \in \mathscr{F}$, any integer $m \geq 4$ and any $c \in \mathcal{C}_0$, we have $d_{\mathcal{C}}(f^m(c), c) \geq 4$. Furthermore, for any $c \in \mathcal{C}_0$ that is non-trivial on $\tilde{S}$, there is $f \in \mathscr{F}$ such that $d_{\mathcal{C}}(f^4(c), c) = 4$.*

References

[1] Farb, B., Leininger, C., & Margalit D., *The lower central series and pseudo-Anosov dilatations.* Amer. J. Math. 130 (2008), 799–827.

[2] A. Fathi, *Dehn twists and pseudo-Anosov diffeomorphisms.* Invent. Math. 87 (1987), 129–151.

[3] Fathi, A., Laudenbach, F., Poenaru, V., *Travaux de Thurston sur les surfaces, Seminaire Orsay,* Asterisque, 66-67, Soc. Math. de France, 1979.

[4] Masur, H., & Minsky, Y., *Geometry of the complex of curves I: Hyperbolicity.* Invent. Math. 138 (1999), 103–149.

[5] Kra, I., *On the Nielsen-Thurston-Bers type of some self-maps of Riemann surfaces.* Acta Math. 146 (1981), 231–270.

[6] Wang, S., Wu, Y, & Zhou, Q., *Pseudo-Anosov maps and simple closed curves on surfaces.* Math. Proc. Camb. Phil. Soc. 128 (2000), 321–326.

[7] Zhang, C., *Pseudo-Anosov maps and pairs of filling simple closed geodesics on Riemann surfaces.* 2012.

[8] Zhang, C., *On distances between curves in the curve complex and point-pushing pseudo-Anosov homeomorphisms.* 2012.

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