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A MATRIX APPROACH TO PLOTTING SECOND-ORDER DIFFERENTIAL EQUATIONS USING LSIM AND ODE23 MATLAB FUNCTIONS

SPYROS ANDREOU, JONATHAN LAMBRIGHT, LEMMA MULATU
*DEPARTMENT OF ENGINEERING TECHNOLOGY AND MATHEMATICS
SAVANNAH STATE UNIVERSITY, SAVANNAH GA 31404*

A MATRIX APPROACH TO PLOTTING SECOND-ORDER DIFFERENTIAL EQUATIONS USING LSIM AND ODE23 MATLAB FUNCTIONS

Spyros Andreou, Jonathan Lambright, Lemma Mulatu
Department of Engineering Technology and Mathematics
Savannah State University, Savannah GA 31404

Abstract: In this experimentation, a differential equation is converted into a matrix form in order to satisfy the requirements for the functions being used in MATLAB. A second-order differential equation is converted into two first-order differential equations. This method requires knowledge of Linear Algebra; however, a lot of information is provided regarding the system such as stability, physical properties and an insight of its behavior after some time giving the designer the opportunity to design other parts for the system such as controllers or observers. The classical methods such as Laplace Transforms will not provide all this information. The case study is a circuit with two energy stored elements (capacitor or inductor) to fully observe the results of the circuit's second-order governing differential equation in either output voltage or current terms. LSIM is a function of MATLAB used to simulate time response of LTI (Linear Time Invariant) models to arbitrary inputs. LSIM plots the time response of the LTI state model (ss in MATLAB) to the input signal described by u (input) and t (time). ODE23 is a function of MATLAB used to solve non-stiff differential equations in the first-order and second-order.

Introduction

The purpose of this research is to understand the link between mathematical equations and the flow and functionality of electricity. The information gathered will aid in the comprehension of the behavior of electricity when governed by the initial differential equation. A good reference for the differential equations is provided in [1]. Further applications can be found in a textbook of control systems [2]. The software MATLAB [3] is utilized to furnish our computing requirements for the complex calculations. MATLAB, which stands for MATrix LABORatory, was created in the late 1970s by Cleve Moler, then chairman of the computer science department at the University of New Mexico. MATLAB has a lot of built in functions making it very attractive to the engineers who wish not to use other higher level programming languages. The simplest way to execute a MATLAB code is to type it in at the prompt, >>, in the Command Window, one of the elements of the MATLAB Desktop. In this way, MATLAB can be used as an interactive mathematical shell. Sequences of commands can be saved in a text file, typically using the MATLAB Editor, as a script or encapsulated into a function, extending the commands available. This paper is divided into two parts the first one is theoretical and the other one is a case study of a second-order electrical circuit.

Theoretical part

The second-order differential equation, $y'' + 5y' + 6y = e^t$, excited by an exponential function will be the theoretical component for the study where the variable y is dependent on time. It will be converted into a matrix form of a system of two first-order differential equations

to satisfy the LSIM function. The LSIM function is studied before utilized by typing (>> is the prompt sign for MATLAB) >> help LSIM in MATLAB. The input equation of this matrix form is $\dot{X} = AX + Bu(t)$ and the output equation is $Y = CX + D$. X is a vector consisting of x_1 and x_2 called the states of the system. This can be done using the following transformation. Let $y = x_1$ and $\dot{y} = x_2$. Differentiating them, it can be obtained $\dot{y} = x_1 = x_2$ and $\dot{y} = x_2 = -6x_1 - 5x_2 + e^t$. Now putting them into a matrix form we get $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t$. Moreover, let the output equation be $Y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and the matrix $D = [0]$ for simulation purposes.

As described above the following program shown in figure 1 is written in MATLAB to simulate the second order differential equation.

LSIM PROGRAM

```
%Differential equation  $y'' + 5y' + 6y = e^t$ 
A=[0 1;-6 -5]
B=[0;1]
C=[1 1]
D=[0]
System=ss(A,B,C,D); %state model
X0=[1 1]; %initial conditions
t=[0:0.01:2]; %time elapsed in seconds
u=1*exp(t); %exponential input
[y,T,x]=lsim(system,u,t,x0);
subplot(2,1,1), plot(T,x(:,2))
xlabel('Time in seconds'), ylabel('state x1')
%plot of a hypothetical output
Subplot(2,2,3), Plot(T,y)
Xlabel('Time in seconds'), ylabel('output y')
```

Figure1. The MATLAB code for the Simulation

RESULTS OF THIS SIMULATION

The following three plots are the results of this simulation. The top two plots are the graphs of states x_1 and x_2 respectively varying with time to an exponential input. The third plot is

the output z which is the sum of $x_1 + x_2$ shown in figure 2.

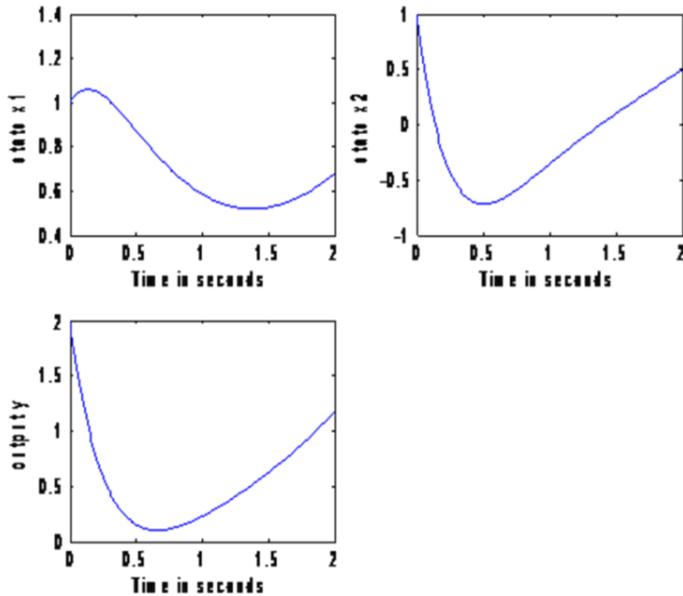


Figure2. Simulation results.

CASE STUDY

The case study is an RLC circuit which is governed by a second order differential equation since it is a 2nd order circuit. The circuit is shown in figure 3 taken from [4]. A very good reference in circuit analysis so that the reader can consult regarding the analysis of the circuit shown in figure 3 is given in [5]. It consists

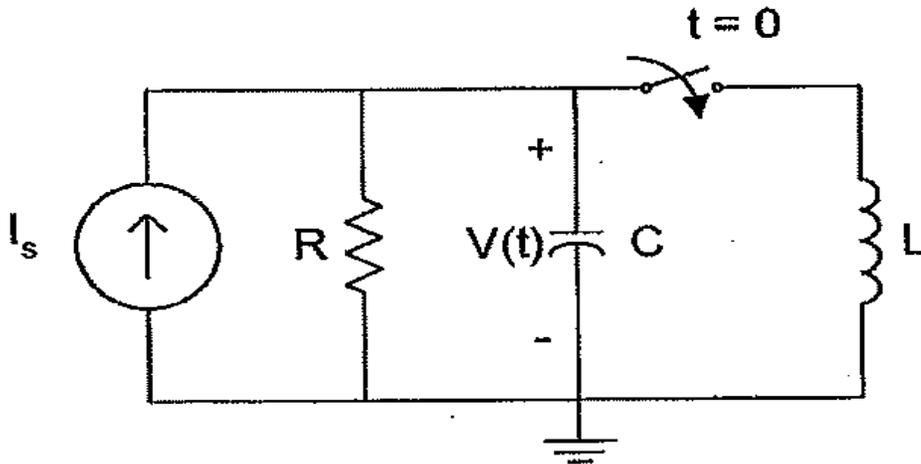


Figure3. An RLC circuit (2nd order)

of a power supply, a switch which closes at $t = 0$, a current source whose value is 2 A (amperes), a resistor $R = 10 \Omega$, an inductor $L = 1/32$ H and a capacitor $C = 50 \mu\text{F}$. Simple analysis shows that at $t < 0$ (switch is open) the voltage across the capacitor, $V(t)$ as shown in the figure 3, is 20

V (volts) and the current across the inductor namely I(t) is 0 A being the initial conditions for the simulation. Utilizing the theory from the previous part, let $x_1(t) = I(t)$ and $x_2(t) = V(t)$, the matrix

equation is obtained being $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} I_S$. The interest here is to see the behavior

of the current across the inductor and the voltage across the terminals of the capacitor. The following MATLAB code consisting of the function *diffa* along with the main program is illustrated in figure 4.

```
%case of an RLC circuit
%solution of second-order differential equation
%the function diffa(x,y) is created to evaluate the diff. equation. The function is as follows:
function xdot = diffa(t,x)
xdot=zeros(2,1); % a column vector needed for the ode function
is=2;
C=50e-6;L = 1/32;R = 10; %microfarad, Henry and ohms.
xdot(1) = (1/L)*x(2);
xdot(2) = (1/C)*(is - x(1))-(1/(R*C))*x(2);
end
%main program
t0 = 0; %in seconds
%final time
tf = 0.05; %in seconds
%initial conditions, inductor current is 0 Amps and capacitor voltage is 20 volts
x0=[0,20];
[T,x] = ode23(@diffa,[0 0.05],[0, 20]);
%x1 represents the inductor current
%x2 represents the voltage current
subplot(2,1,1), plot(T,x(:,1))
xlabel('Time, s'), ylabel('inductor Current, A')
subplot(2,1,2), plot(T,x(:,2))
xlabel('Time, s'), ylabel('capacitor voltage, V')
```

Figure4. MATLAB code for the RLC circuit

The comments in the code are indicated by the percent sign so that the reader has an easier understanding of each command. Using the MATLAB command `>> help ode23`, the MATLAB function is studied first and then utilized. Moreover, this function is calling the function *diffa* written by the users. It should be noted that if another high-level programming language was used to perform these simulations, it would have taken a lot of time. These simulations demonstrate how powerful and useful MATLAB is. The simulation results are illustrated in figure 5. The current through the inductor and the voltage across the capacitor hold physically and as expected.

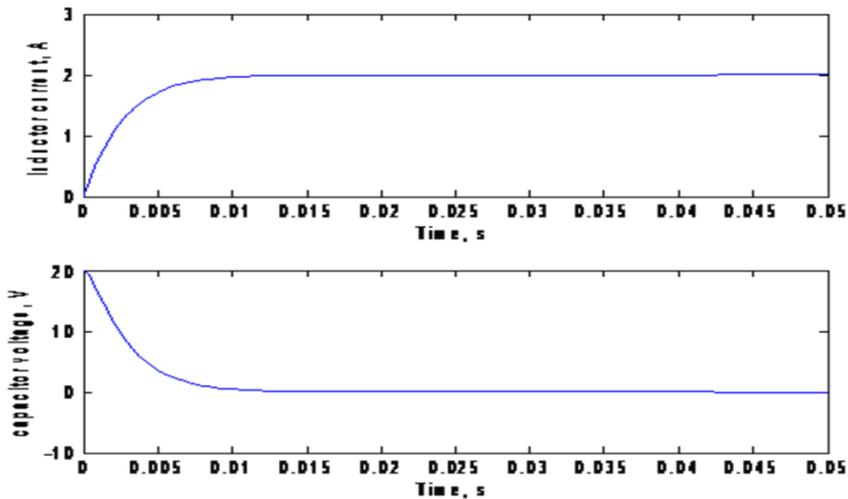


Figure5. Simulation results

CONCLUSION

In this endeavor second-order differential equations were solved using Linear Algebra methods; that is, reducing it to two first-order differential equations and forming input and output matrix equations. This method works for n^{th} -order differential equations. Doing this, the MATLAB software can be utilized with its two built-in functions, LSIM and ODE23, performing the simulations and obtaining excellent results. These results hold physically as well. This work demonstrates also that MATLAB is a powerful tool for engineers for obtaining fast and reasonable results. It would have been very difficult to write programs in other high-level programming languages to accomplish this work.

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