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SIMULATE POKER PROBABILITY OF PARTIAL DECK

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Abstract

Poker is a very popular gambling game in Casinos. Except professionals, most Poker players lost money without applying the proper probability. This STEM project is to study the Poker probability by using partial deck. To simplify probability simulation, partial deck was used and which can increase the matching probability significantly on some higher ranked patterns such as "Full House" and "Straight". However, partial deck can also decrease the matching probability on some patterns such as "Flush" and "Nothing". The ranking of matching patterns is also changed with Partial Deck as compared to the Full Deck. The authors have used combination formula and derive the general formulas of matching probability for each matching pattern. This is a great STEM project to integrate STEM Learning and Probability through a very practical Poker Game.

Keywords
Java, Statistics, Poker Probability, Monte Carlo Simulation, Partial Deck

1. Introduction and Literature Research

Most Poker players lost Money in Poker Gambling since they played blind gambling without applying the poker probability and assess their risk on each play. The objective of this paper is to use JAVA to simulate Poker Probability and study Sample Size effect on Statistics and decision making. The project scope is for learning purpose, not for gambling purpose. Authors used partial deck (9, 10, J, Q, K, A) of 24 cards to simplify JAVA poker simulation. Figure 1 has listed the rankings of different matched patterns for the full deck (52 cards) scenario. The full deck poker for 6 to 7 random cards is very popular in most Poker tournament [1, 2]. Several research papers have demonstrated and simulated the poker probability by using Monte Carlo Simulation [3], Evolutionary Computing [4] and Artificial Intelligence [5]. There is also an US Patent [6] studied the partial deck on Royal Flush probability. In this paper, the authors will study the Poker Probability on the 24-cards Partial Deck and use JAVA Monte Carlo Simulation on a special case study to verify the winning probability between two players. The ranking of Partial Deck may be different from the Full Deck. In Figure 1, the Poker Hand Rankings are displayed from Royal Flush to High Cards (Nothing). The matching probability of these patterns are also shown in Figure 2. It's less than 1% probability chance to match the "Straight" or higher patterns for a full deck game. The Poker players will have to analyze the complex situations in real time quickly in order to have a reliable judgement. Therefore, this STEM paper will use the partial deck to simplify the situations and to simulate the Poker probability of winning the higher ranked patterns. The objective is to adjust the number of cards and calculate the expected matching probability.

Figure 1: Poker Hand Rankings

	Full Deck		
	Trial	**Event**	**Probability**
Royal Straight	C(52, 5) 2,598,960	C(4,1)	0.000%
Straight Flush		C(4,1)*C(9,1)	0.001%
Four of a Kind		C(13,1)*C(12,1)*C(4,1)	0.024%
Full House		C(13,1)*C(12,1)* C(4,3)*C(4,2)	0.144%
Flush		C(4,1)*C(13,5)- C(4,1)*C(10,1)	0.197%
Straight		C(10,1)*[C(4,1)^5- C(4.1)]	0.392%
Three of a Kind		C(13,1)*C(12,2)* C(4,3)*C(4,1)*C(4,1)	2.113%
Two Pair		C(13,2)*C(11,1)* C(4,2)*C(4,2)*C(4,1)	4.754%
One Pair		C(13,1)*C(12,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)	42.257%
Nothing		[C(13,5)-10]* [C(4,1)^5-4]	50.118%

Figure 2: Full Deck Matching Probability.

II. Study Partial Deck Probability

Authors used partial deck (9, 10, J, Q, K, A) of 24 cards as an example to simplify the Poker probability… Partial Deck can increase the matching probability especially on higher ranked patterns such as Four of a Kind, and Full House. Partial Deck Poker may also simplify probability simulation process concentrated on higher ranked patterns which may be critical for Poker Players in real time decision making on each betting move.

2.1 Probability Comparison of Four of a Kind

When the Poker Cards have been reduced from the Full Deck (52 Cards) to Partial Deck (24 Cards), as shown in Figure 3, the trial space is reduced by 60X factor from Combination (52, 5) to Combination (24, 5). The event of "Four of a Kind" is also reduced by 5X from 624 to 120. The even matching probability has been increased by 12X from <0.0001% to 0.002%.

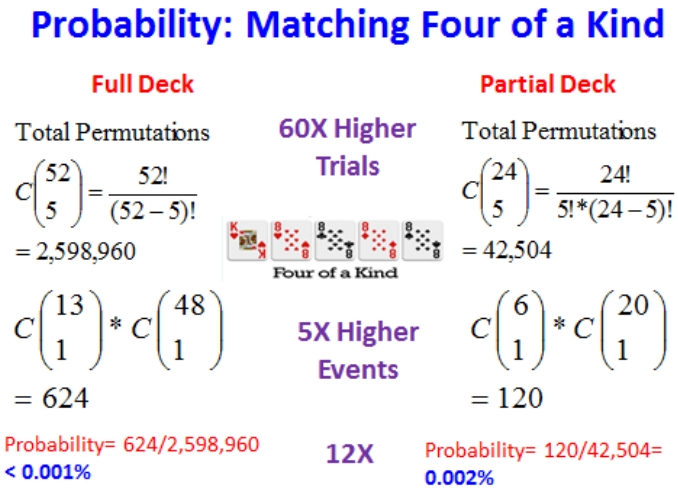

Figure 3. Probability Comparison: Four of a Kind.

2.2 Odds Ratio Comparison of Full Deck vs. 24-Cards Partial Deck

To extend the probability change from Full Deck to Partial Deck, as shown in Figure 4, the odds ratio has been derived. Partial Deck has significantly increased the matching probability[6] except for "Flush" and "Nothing" Cases. These calculations are based on between Full Deck and Partial Deck of 24 Cards. The ranking of matching probability is also changed. For example, for 24-cards partial deck, the probability of matching "Flush" is lower than the probability of matching "Full House", same as "Nothing" lower than "One Pair". This ranking order change is critical if the Poker Game has used only partial deck.

	Full Deck			24 Partial Deck			
	Trial	Event	Probability	Trial	Event	Probability	Ratio
Royal Straight	C(52, 5) 2,598,960	C(4,1)	0.000%	C(48,5) 42,504	C(4,1)	0.009%	61.1
Straight Flush		C(4,1)*C(9,1)	0.001%		C(4,1)*C(1,1)	0.009%	6.5
Four of a Kind		C(13,1)*C(12,1)*C(4,1)	0.024%		C(6,1)*C(5,1)*C(4,1)	0.282%	11.7
Full House		C(13,1)*C(12,1)* C(4,3)*C(4,2)	0.144%		C(6,1)*C(5,1)* C(4,3)*C(4,2)	1.694%	11.8
Flush		C(4,1)*C(13,5)- C(4,1)*C(10,1)	0.197%		C(4,1)*C(6,5)- C(4,1)*C(2,1)	0.038%	**0.2**
Straight		C(10,1)*[C(4,1)^5- C(4.1)]	0.392%		C(2,1)*[C(4,1)^5- C(4,1)]	4.800%	12.2
Three of a Kind		C(13,1)*C(12,2)* C(4,3)*C(4,1)*C(4,1)	2.113%		C(6,1)*C(5,2)* C(4,3)*C(4,1)*C(4,1)	9.034%	4.3
Two Pair		C(13,2)*C(11,1)* C(4,2)*C(4,2)*C(4,1)	4.754%		C(6,2)*C(4,1)* C(4,2)*C(4,2)*C(4,1)	20.327%	4.3
One Pair		C(13,1)*C(12,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)	42.257%		C(6,1)*C(5,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)	54.207%	1.3
Nothing		[C(13,5)-10]* [C(4,1)^5-4]	50.118%		[C(6,5)-2]* [C(4,1)^5-4]	9.599%	**0.2**

Figure 4. Odds ratio comparison of Full Deck to Partial Deck.

2.3 Derive General Partial-Deck Formula

As shown in Figure 5, authors further use Combination and Conditional Probability to calculate the expected matching probability of the Partial Deck in General. "m" number is how many different card numbers chosen in the partial deck. For example, when we choose "9,10,J,Q,K,A", "m" =6, and total 4m=24 cards. In Figure 5, the event space is also changed depending "m" in partial deck. The probability of matching any particular pattern is also depending on "m". Authors have successfully built the general Partial-Deck Probability Formula.

	4*m Partial Track		
	Trial	Event	Probability
Royal Straight	C(4m,5)	C(4,1)	C(4,1)/C(4m,5)
Straight Flush		C(4,1)*C(m-5,1)	C(4,1)*C(m-5,1)/C(4m,5)
Four of a Kind		C(m,1)*C(m-1,1)*C(4,1)	C(m,1)*C(m-1,1)*C(4,1)/C(4m,5)
Full House		C(m,1)*C(m-1,1)*C(4,3)* C(4,2)	C(m,1)*C(m-1,1)*C(4,3)* C(4,2)/C(4m,5)
Flush		C(4,1)*C(m,5)-C(4,1)*C(m-4,1)	[C(4,1)*C(m,5)-C(4,1)*C(m-4,1)]/C(4m,5)
Straight		C(m-4,1)* [C(4,1)^5-C(4,1)]	C(m-4,1)* [C(4,1)^5-C(4,1)]/C(4m,5)
Three of a Kind		C(m,1)* C(m-1,2)*C(4,3)*C(4,1)*C(4,1)	C(m,1)* C(m-1,2)*C(4,3)* C(4,1)*C(4,1)/C(4m,5)
Two Pair		C(m,2)*C(m-2,1)*C(4,2)*C(4,2)*C(4,1)	C(m,2)*C(m-2,1)* C(4,2)*C(4,2)*C(4,1)/C(4m,5)
One Pair		C(m,1)*C(m-1,3)*C(4,2)*C(4,1)*C(4,1)*C(4,1)	C(m,1)*C(m-1,3)*C(4,2)* C(4,1)*C(4,1)*C(4,1)/C(4m,5)
Nothing		[C(m,5)-(m-4)]*[C(4,1)^5-C(4,1)]	[C(m,5)-(m-4)]*[C(4,1)^5-C(4,1)]/C(4m,5)

Figure 5. General Matching Probability Formula of Partial Deck

3. Results

In order to investigate and evaluate the matching probability of Partial Deck at different "m" size, the authors have compared the matching probability of "Flush", "Full House", and "Straight" with "m" number from 1 to 13 (4 cards to 52 cards) as shown in Figure 6. When "m" is too small, some matching probabilities are not available (NA listed in Figure 6) to form such particular patterns. Also, the trending of the "m" size effect is quite different each other. For example, the "Flush" matching probability is increasing when increasing the "m" number. The "Full House" and "Straight" are opposite. This is reasonable. We need more cards available to pick five cards to form "Flush (same type of cards). In the other way, we want to reduce the variety of card numbers to form "Full House". Based on the partial deck probability, we can adjust the odds of poker game by changing the "m" in partial deck.

Trial			Flush				Full House				Straight		
Trial=C(4m,5)			Event=C(4,1)*C(m,5)-C(4,1)*C(m-4,1)			Matching Probability	Event=C(m,1)*C(m-1,1)*C(4,3)*C(4,2)			Matching Probability	Event=C(m-4,1)*[C(4,1)^5-C(4,1)]		Matching Probability
m	4m	Trial	C(m,5)	c(m-4,1)	Event	Event/Trial	C(m,1)	c(m-1,1)	Event	Event/Trial	C(m-4,1)	Event	Event/Trial
1	4	NA	NA	NA	NA	NA	1	NA	NA	NA	NA	NA	NA
2	8	56	NA	NA	NA	NA	2	1	48	85.71%	NA	NA	NA
3	12	792	NA	NA	NA	NA	3	2	144	18.18%	NA	NA	NA
4	16	4368	NA	NA	NA	NA	4	3	288	6.59%	NA	NA	NA
5	20	15504	NA	NA	NA	NA	5	4	480	3.10%	1	1020	6.58%
6	24	42504	6	2	16	0.04%	6	5	720	1.69%	2	2040	4.80%
7	28	98280	21	3	72	0.07%	7	6	1008	1.03%	3	3060	3.11%
8	32	201376	56	4	208	0.10%	8	7	1344	0.67%	4	4080	2.03%
9	36	376992	126	5	484	0.13%	9	8	1728	0.46%	5	5100	1.35%
10	40	658008	252	6	984	0.15%	10	9	2160	0.33%	6	6120	0.93%
11	44	1086008	462	7	1820	0.17%	11	10	2640	0.24%	7	7140	0.66%
12	48	1712304	792	8	3136	0.18%	12	11	3168	0.19%	8	8160	0.48%
13	52	2598960	1287	9	5112	0.20%	13	12	3744	0.14%	9	9180	0.35%

Figure 6 Demonstrate matching probability vs. "m" sample size.

Authors further plot the trending of matching probability vs. "m" size in Figure 7 to illustrate Figure 6 observations visually. There is ranking switch between "Flush" and "Full House" when "m"= 12.

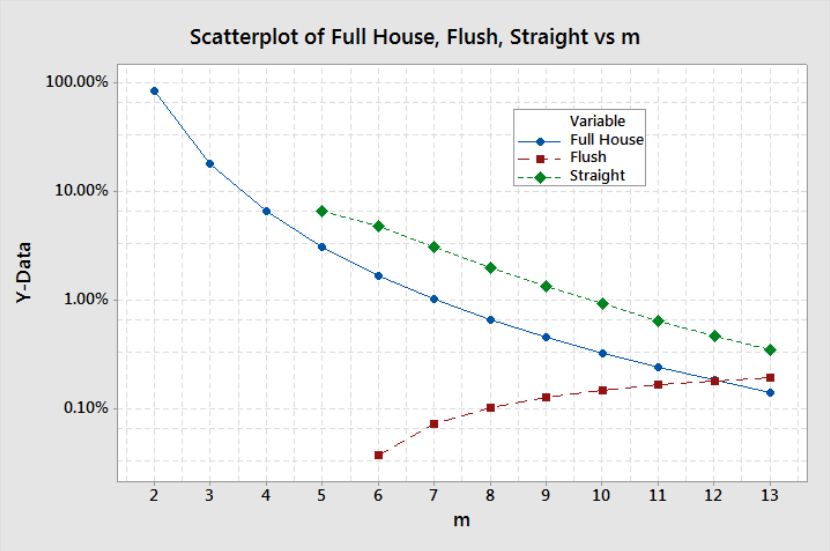

Figure 7 Trending scatterplot of matching probability vs. "m" size.

4. Conclusions

The main conclusion of this paper is for Poker Players to concentrate more on the higher ranked patterns when partial deck size is getting smaller. Simplifying the poker probability in real time can be a big deal over pure guessing or gambling. The ranking will be different when the partial deck size is getting smaller (Flush vs. Full House). Knowing the Poker probability and ranking may take a huge advantage when the Partial Deck "m" size is getting smaller. However, when m is too small, most cards will be known, then the uncertainty is much reduced. This paper may guide the Casino and innovate the Poker Games to attract more Poker players due to a higher chance of winning bigger hands in Poker Games. Poker players may feel more excited and may bet money wisely. This is a good STEM project for High School students to apply their AP Statistics knowledge in a real case study. Thanks for IEOM for providing such learning opportunity in the Bogota Conference.

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