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STUDENTS IN ACTION: THE GREAT ICOSAHEDRON CHALLENGE

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Students in ACTION: The Great Icosahedron Challenge

Synopsis:

This presentation illustrates with pictures and words the trials and tribulations of a group of six freshman math and science education majors at Bowling Green State University as they embarked on attempting to build a 3D no-sew icosahedron quilt. Starting with an ill-defined idea and particular constraints, the students had to investigate the necessary mathematics, develop appropriate mathematical models and translate the mathematics into an actual 3D mathematical art piece.

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Keywords: art, mathematics, collegiate, exploration, three-dimensional

Subject classification codes: include these here if the journal requires them



Declarations and Directions

Mathematics seeks to describe reality by looking at the logical interrelationships between concepts. Through art, we experience reality in ways not directly accessible to reasoning, but which we find intuitively meaningful... Both try to express fundamental 'truths' about the nature of reality, seeking structure and symmetry within the complex universe in which we find ourselves. [4, paragraph 7]

For the past several semesters, I have been working with groups of freshman Math and Science Education students with various mathematical backgrounds and introducing them to research problems that extend their knowledge of mathematics and start to see mathematical ideas in a different light. This paper seeks to describe the various trials and tribulations that a particular group of students encountered when exploring the intersection of mathematics and art. The freshman research group of 2013 began by looking at various mediums that mathematical art is expressed and then embarked on the development of mathematical pieces that integrate pattern and precision. We then used the resulting pieces to explore the deeper mathematical underpinnings of the pieces and prove those mathematical connections. Finally, we discussed ways to share the mathematics with students. In doing so, the hope was that the students would (1) gain a better appreciation of the world around us and the mathematics which underlies our descriptions of that world; (2) explore various types of art and the mathematics inherent to the depiction and expression; (3) construct various pieces of mathematical art that can be used as teaching tools in the mathematics classroom; (4) explore some higher level mathematics; and (5) build brief vignettes explaining the underlying mathematics accessible to middle, high school or college students..

This paper will focus on one of the major projects that the research group explored – making a no-sew icosahedron quilt. In particular, we will focus on the development, design, dedication, decision-making, and display. First off, let us explain that the idea of building a 3D icosahedron using no-sew quilting techniques was somewhat an abstract concept that none of us knew if it could be done when we started into the project. The students had engaged in no-sew quilting similar to the designs described in [5]. The no-sew quilts in [5] exceed the ‘quilts’ described by [2, 8, 9] since those authors merely suggested colored construction paper or fabric being affixed to

poster board. In particular, the no-sew quilts in [5] have the dimensionality and finished quality of real quilts. But could a three-dimensional no-sew quilt be built and why would one care to do so?

Clearly, quilt patterns have distinct connections to triangle congruence, reflections, and transformations [2]. But the extent of the potential benefits of incorporating the investigation of quilting in the mathematics classroom does not stop there. Paznokas argued that there was much more that can be drawn from such examinations of quilting when she said, ‘Not only can quilts help teach multicultural history, literacy, and art, but quilting also provides a perfect and enjoyable tool for teaching mathematics concepts’ [6, p. 250]. Although [2, 6] were focused on mathematical connections such as symmetry, transformations, ratios, patterns, tessellations, and fractions, quilts can also be used to explore higher-level mathematical ideas that touch concepts at the high school level and even beyond. For instance, the Quaternion Quilts described by [1] certainly extend beyond high school mathematics to demonstrate group theoretic relationships from abstract algebra and the fractal quilts designed by [7] provide a venue to talk about chaos theory. Except for some tufting placed within a quilt, the quilts that have been developed and discussed have focused on primarily two-dimensional structures. This three-dimensional mathematical art project sought to test the ingenuity and problem-solving ability of these students. In doing so, we were able to document the investigation what mathematical ideas would students need to bring to bear as they embarked on an ill-defined mathematical three-dimensional art project.

Definitions

‘An icosahedron is a polyhedron which is a 3D object and this is made up of 20 equilateral triangles. So that’s why our research question was, we want to make

the largest twenty triangles so we can make the largest possible icosahedron. And why were we making this? Because Dr. Meel wanted to challenge us to see if we could do it, which we did. And we also made an awesome piece of art using mathematics.’ ~ Alli

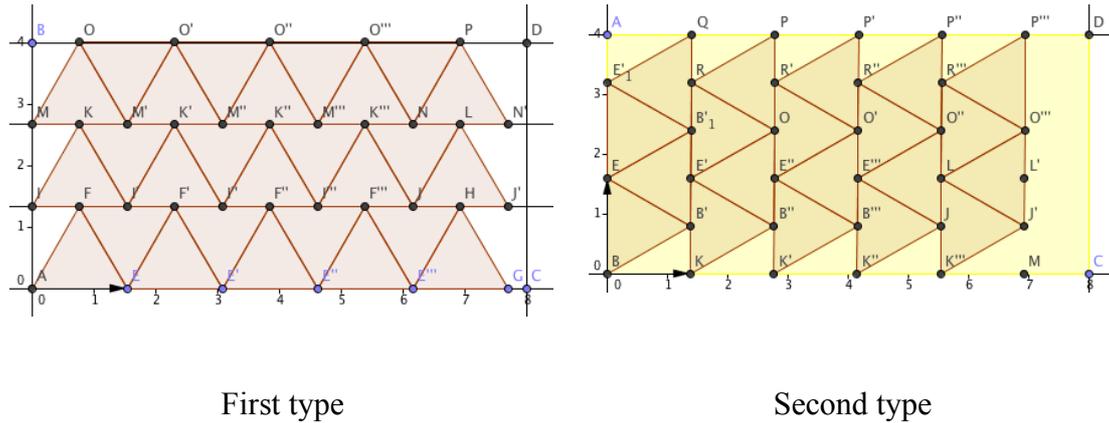
Since an icosahedron is formed by twenty faces made of equilateral triangles, the students in the research group settled on the following main research question: ‘How do we make the Largest 20 triangles with minimal waste on a 4’ x 8’ sheet of foam board?’ The students in the research group were provided a 4’ x 8’ sheet of foam board and had to figure out how to minimize waste as much as they could in order to construct the largest triangles. It should be mentioned that the phrase ‘minimal waste’ was a relative term to them and this was not an exercise in proving that we had achieved a true minimum as would be the case of a packing problem but rather that the dissection of the 4’ x 8’ foam board would be somewhat efficient and regular in structure.

Development

‘We had to figure out if we could fit the twenty triangles on the foam board. But not just if we could do it; but [also] how we would do it. So, we made scale models on pieces of paper to see how we would orient the triangles. Some of us had it horizontally, some vertically. Some people had 17 triangles and some people had 27.’ ~ Maria

The students’ initial sketches to build the 20 triangles fell into two main image types (see figure 1). The first attempt focused on vertical partitioning of the 4’ x 8’ sheet.

Figure 1: Partitioning of the 4’ x 8’ foam board



This partitioning of the rectangle in the first type resulted in 27 regular equilateral triangles meaning that we would have considerable waste since only 20 would be needed. In contrast, the second type focused on horizontal partitioning of the space. Here twenty equilateral triangles are produced which are clearly larger than those generated by vertical partitioning. However, the question was raised about whether we could do better.

Design

‘But eventually, we came up with how to fit twenty on there. And we want to find out how to make the triangles as large as we could, so we used a program called Geogebra, and we found that if we put the triangles on an angle ... we could make the triangles bigger’ ~ Maria.

Over time, the students in the research group were able to recognize that a variation on the horizontal partitioning would reduce the waste and increase the size of the equilateral triangles.

Figure 2: Diagonal Partitioning of the 4' x 8' foam board



Through their analysis (see figure 4), the students were able to determine that

$$(1) \theta + \phi = \frac{\pi}{6},$$

$$(2) J = 4\tan\phi,$$

$$(3) p = \frac{\sqrt{3}}{2} x * \sec\phi,$$

$$(4) r = \frac{16\sqrt{3} \cos \phi - 8\sqrt{3} \sin \phi}{30} - \frac{\sqrt{3}}{2} \left(\frac{16\sqrt{3} \cos \phi - 8\sqrt{3} \sin \phi}{15} \right) * \tan \phi, \text{ and}$$

There is an old saying that ‘many hands make light work’ but in this case, many hands and many brains were necessary to check and double-check that all the lines were correctly struck.

Dissecting

‘Once we had all of our measurements laid out onto the foam, we knew that we had to cut into the foam at an angle so that we could put the pieces of the 3D object together. So we calculated [that] a 60-degree angle is what we needed to cut into it. And we created a device that would cut into the foam board.’ ~ Liz

There was one other element of the implementation process that required the research group to further investigate the characteristics of the icosahedron. Each of the equilateral triangles would need to be bevel cut so that they would fit together. The problem posed to the group was to determine the angle of the bevel and then provide directions as to what tool (see figure 7) I would need to make in order to cut the triangles so they would fit together.

Figure 7: Preliminary sketch of the beveling tool

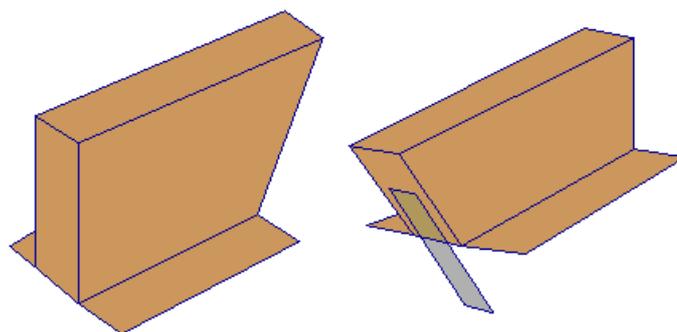
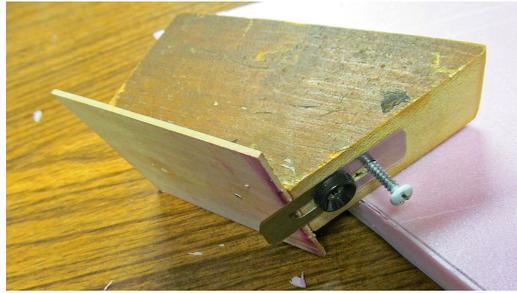


Figure 8: The constructed beveling tool



This tool was then used to dissect the 4' x 8' foam board while cutting bevels into all of the equilateral triangles in anticipation of putting all of them together to form the icosahedron. It is one thing to know what has to happen and completely another thing to design a new tool to make the design come to fruition.

Display

'Our last part was using our creativity. We each got three triangles and Dr. Meel got two, that was nice for him. He got to be creative like us. We used x-acto knives, ... to score each triangle into different patterns. We were all different and creative in our own ways and we also used the x-acto knives to score it... We made twenty in the end and it created our icosahedron. In the end, to make it come together we used spray foam and as Liz said, when we came up with our angles, it came together quite well but the spray foam was used so that they could stick together and then we used packing tape which, luckily you cannot see, because we also covered that with fabric. So it just looks like one big icosahedron.' ~ Sherri

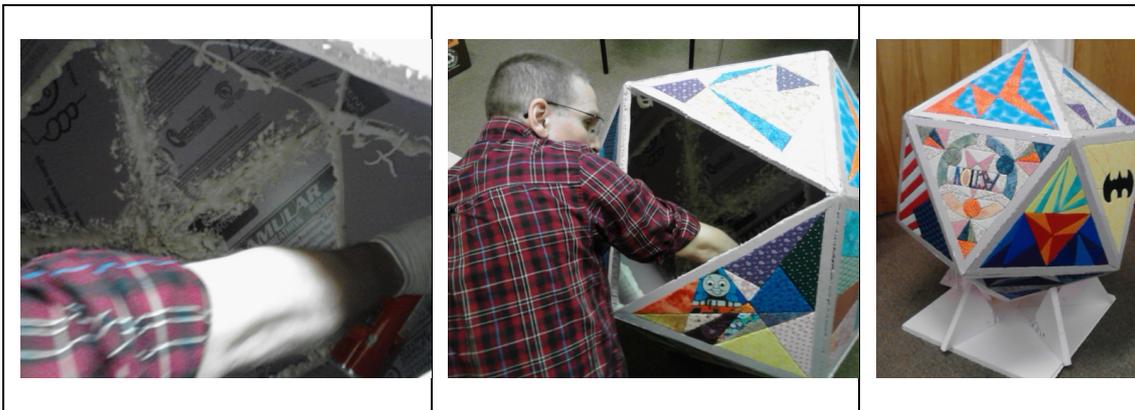
Once the 4' x 8' foam board was partitioned and the triangles were beveled, the students of the research group embarked on quilting their three triangles (see figure 9).

Figure 9: Students quilting their icosahedron triangular faces



One of the more difficult elements of building the 3D icosahedron quilt was supporting the structure as it was being constructed. However, with some ingenuity and the power of Great Stuff spray foam (<http://greatstuff.dow.com/>) on the inside and packing tape on the outside, we were able to support and connect the various faces (see figure 10).

Figure 10: Using spray foam to provide internal support to the icosahedron quilt



Once all the triangular faces were in place, the final task was to cover all of the taped seams with fabric.

Dismissal

This project reaches beyond the ideas presented in [5] by building something that had never been attempted before. The idea of a three-dimensional no-sew quilt was borne out of the rigid medium by which no-sew quilts are made. The fact that foam board is used provides structure and a framework that is not constrained by the flexible fabric

which traditional quilting employs. The question was, could a group of freshman Math and Science Education majors at Bowling Green State University take such an ill-defined proposition and investigate the necessary mathematics, develop appropriate mathematical models and translate those mathematics into an actual 3D mathematical art piece.

Clearly, through the words and images shown in this paper, the answer is a resounding yes! We recognize, as did [5] that only a brave, or perhaps foolish, math teacher at the high school or middle school level would directly translate this activity into their classroom when it includes sharp x-acto knives. The warnings and accommodations mentioned in [5] do hold for this activity. In addition, we need to mention that with the inclusion of Great Stuff™ spray foam, one needs to be especially vigilant because it can ruin carpet, clothing, and fabric if not handled carefully.

This paper has demonstrated the multi-faceted thinking that students need to engage in when approaching such an ill-defined proposition. The learning and insights gained are invaluable as they prepare to be middle school or high school math or science teachers. Being engaged in a long-term, multi-semester project forced these prospective teachers to grapple with not only the ‘what’ of mathematics but also the ‘how’ and ‘why’. Getting students to envision mathematical concepts in a variety of ways is essential to good teaching and enhances student learning. The project of building a 3D no-sew icosahedron set the stage for investigations and challenges by providing access to strong visual images that illustrate important mathematical concepts. We think the saying should be changed to: A quilt is worth a thousand words but a 3D icosahedron quilt is worth ten thousand!

Figure 11: Students holding the icosahedron quilt



References: see the journal's instructions for authors for details on style

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Figure 12: Net of the quilt icosahedron showing all twenty of the quilted sides

