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ALGEBRA PROFICIENCY IN CALCULUS: A LOOK AT ENGLISH LANGUAGE LEARNERS

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Algebra Proficiency in Calculus: A Look at English Language Learners

Synopsis:

Algebra proficiency of ELLs in first semester calculus at a small private liberal arts college, with a student body of approximately 50% foreigners and ELLs, was studied. Inferential statistics were used to analyze the data. The results indicate that ELLs, including East Asians, are extremely weak in algebra at the beginning of calculus. However, by the end of first semester calculus, ELLs outperformed native English speakers and reached the algebra proficiency benchmark of 75%.

Algebra Proficiency in Calculus: A Look at English Language Learners

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ABTRACT

Proficiency in both algebra and calculus of students in first semester calculus at a small private liberal arts college, with a student body of approximately 50% foreigners and ELLs was studied. Due to the high proportion of ELLs meticulous attention to definitions, mathematical terminology, use of functional language, and coupling functional language with graphics were carefully employed while teaching. A one sample proportion test, one sample t-test, and independent t-test were used to analyze the data. The results indicate that the vast majority of students are *extremely* weak in algebra at the beginning of calculus. By the end of the semester, ELLs reached statistically significant higher levels of algebra proficiency than native English speakers. In addition, the overall average scores (for calculus) of ELLs were higher than the overall average scores of native English speakers but, the difference in the average scores was not statistically significant.

INTRODUCTION

Weak Algebra Skills

Many students entering college are weak in algebra including students majoring in science, mathematics, and engineering (Bailey, Jeong & Cho, 2010; Budny, Bjedov & LeBold, 1998; Jourdan, Cretchley & Passmore, 2007). Between 1966 and 1993, engineering students at Purdue University dropped engineering as their major due to struggles in calculus (Budny, Bjedov & LeBold, 1998). Science departments at Purdue performed studies to determine why so many Purdue students struggle in courses such as physics, chemistry and calculus. The findings indicated weak algebra skills were the underlying problem in all the said sciences courses, including calculus (Budny et al., 1998).

In a study by Jourdan, Cretchley and Passmore (2007), new college students at an Australian university, whose major was science, were found to be weak in algebra. Over 40% of the students were unable to factor basic quadratic expressions and solve simple quadratic equations. Fifty-nine percent of the Australian students were incapable of subtracting two rational expressions and, given $f(x)$, 61% were unable to find $f(x+h)$.

In a study by Orton (1983), 110 calculus students from both high schools and colleges were clinically interviewed. Orton found these students struggled with solving basic algebraic equations. In the process of working calculus problems, Orton found several students were unable to correctly solve the quadratic equation $3x^2 - 6x = 0$. Not only did students make procedural mistakes but, many gave one solution, suggesting several students could not identify the equation as quadratic or they did not understand quadratic equations have two solutions. In other words, many students lacked both procedural and conceptual understanding.

In another study by Martin (2000), which focused on large urban university calculus students' ability to solve related rates problems, the author assumed students had considerable prior experience with solving algebraic equations. The author also assumed students were proficient in algebra. Students did in fact have a decent amount of experience but, Martin found students lacked fundamental algebra skills. Martin's conclusion was students' prior achievement in algebra does not imply algebra proficiency nor sufficient readiness for calculus.

In this study, the vast majority (89%) of students were either math or science majors. The sample consisted of 44% English language learner (ELLs) and about 50% of the entire sample was from East Asia, the Pacific Islands or Central/South America. This study examines algebra proficiency of foreign and ELL calculus students.

Language

Mathematics is a language, even a symbolic language (Barton, 2007; Schleppegrell, 2010). However, until recently, thoughts in math education suggested that drawing on language was secondary for learning mathematics (Schleppegrell, 2010). Today, knowing and interpreting the language of mathematics is seen as a major factor in understanding math (Schleppegrell, 2010).

Relative to ELLs learning calculus, an additional layer is added to the complexity of language. ELLs are learning the language of calculus in a language they are not fluent (English) which means ELLs are using and learning two new languages, the calculus and English (Usiskin, 1996). Because of the lack of algebra proficiency, it could be argued that ELLs are actually learning three languages simultaneously with algebra being the third language.

According to Cohen (2014), when learning a new language (including mathematics) beginners prefer every word be interpreted and defined. To help students understand the language of mathematics, while teaching, instructors should pay close attention to mathematical terminology (Barwell, 2005; Choi et al., 2013; Colker, Toyama, Trevisan & Haertel, 2003). To help reduce challenges for students, and especially ELLs, instructors should clearly and explicitly define terms, even terms that might appear to be every-day words (Roessingh & Douglas, 2012). For example, when teaching ELLs, calculus instructors need to be careful with assuming their students are familiar with common terminology such as "perimeter." Similarly, terms such as differentiate and differential are very similar in pronunciation and should therefore be clearly defined and articulated when teaching. By helping students construct mathematical knowledge through appropriate use of language, students will more accurately understand mathematical meaning (Schleppegrell, 2007). By understanding mathematical meaning students, including ELLs, will be more likely to succeed (Schleppegrell, 2007).

Supplementing Language with Graphics

Graphic displays, which are plentifully used in calculus, are not sufficient for complete understanding of mathematical content (Aguirre-Muñoz, Boscardin, Jones, Park, Chinen, Shin, Lee, Amabisca & Benner, 2006). Functional language must be carefully coupled with graphics to reach a more meaningful level of understanding; this is especially true when teaching ELLs (Aguirre et al., 2006). For example, the rigorous definition of a local maximum is "*f* has a local maximum at $c \in (a,b)$ if $f(c) > f(x)$ for all $x \in (a,b)$." This definition, with the mathematical symbols, is difficult to understand even for native English speakers. To clarify what this definition is saying, after writing the definition, it should be (re)read while simultaneously pointing to each symbol as its associated word is spoken. In this case, while pointing to each corresponding word or symbol, the instructor would read, "The function '*f* 'has a local maximum, or largest value, at *c* which is contained in the open interval (*a*, *b*) if the value of the function at *c* is larger than the value of the function at any other *x*-value contained in the open interval (*a*, *b*)."

To further strengthen students' understanding, a supplemental graphic is crucial (See Figure 1) (Aguirre et al., 2006). While verbalizing the definition yet again, use the graphic to show what *f* is, how *c* is contained in the interval (a,b), and how the largest value of *f* in the interval (*a*, *b*) is located at *c*. Finish by showing (on the graphic) how $f(c)$ is greater than $f(x)$ for every *x* in the interval (a,b).

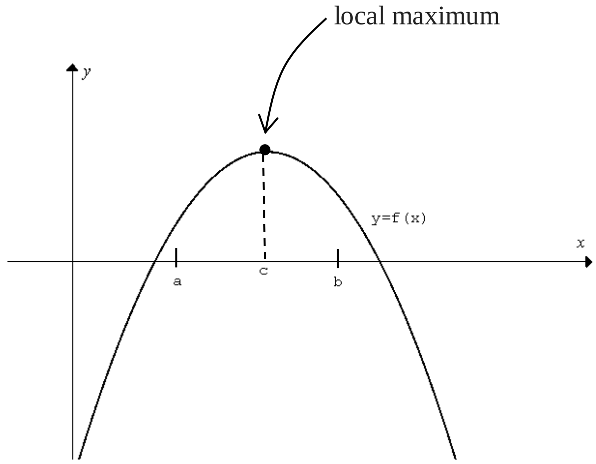

Figure 1. Supplemental graphic.

Skipping Steps

To skip steps in the solution process one must do mental calculations. However, practicing problems over and over is necessary to reach the point where one can do mental calculations and thus skip steps (Roediger & Butler, 2011; Schraw & McCrudden, 2003). As students solve the same problem type over and over, students slowly begins to solve the problem using different mental processes which results in the ability to skip steps (Blessing & Anderson, 1996; Charness & Campbell, 1988; Koedinger & Anderson, 1990). To illustrate the point, when first learning (or relearning) how to solve the quadratic equation, $x^2 - x - 6 = 0$ students need to see every step in the process to understand why the solutions are $x = 3$ and $x = -2$ (Blessing & Anderson, 1996). In other words, students need to see the following:

$$\begin{gathered} x^2 - x - 6 = 0 \\ (x-3)(x+2) = 0 \\ x-3=0, \;\; x+2=0 \\ x=3, \qquad x=-2 \end{gathered}$$

After students have done similar problem types over and over they can visualize in their mind the process of factoring and other algebraic procedures. Through repeated practice, students will begin to use different mental processes and will have the capacity to skip steps because of their increased ability to visualize the written process (Blessing & Anderson, 1996).

Relative to first semester calculus, most students are weak in algebra and are therefore unable to do mental algebra. When instructors skips steps while teaching, ELLs especially have a heavy cognitive load because they are trying to do mental algebra and they are, (1) weak in algebra, (2) struggling with the language (English) in which the subject is taught, and (3) learning the new language of calculus. As a result, when calculus instructors skip algebraic steps, ELL's working memory is overloaded and learning decelerates. Therefore, skipping steps should be avoided in first semester calculus, particularly towards the beginning of the semester when students' algebra proficiency is exceptionally weak. Likewise, skipping steps in the calculus should also be avoided.

Guided Instruction

Incorporating effective verbal explanations with written worked examples results in guided instruction when the instructor serves as a model or coach (Kirschner, Sweller & Clark, 2006; Mayer, 2011). Guided instruction increases overall pedagogical effectiveness (Kirschner, Sweller & Clark, 2006). According to both sociocultural theory and involvement theory, instruction should also include frequent interaction between instructors and students (Astin, 1990; Scott & Palincsar, 2009). For instance, to increase student-faculty interaction, when solving an un-factorable quadratic equation, instructors could ask students, “Why does the next step require the quadratic formula?” The question actively guides and engages students while simultaneously using mathematical terminology. Relative to the calculus, when calculating derivatives of quotients, instructors could ask, “Should we use the product rule or quotient rule for derivatives?” Again, the question guides students while using terminology of calculus.

Pacific Island Students

Achieving educational excellence and equality is a fundamental principle of multicultural education (Bennett, 2001). Furthermore, at the college where this study was conducted, a high proportion of students are from the Pacific Islands. According to the National Commission on Asian American Pacific Islander Research in Education (2010), 51.8% of Samoans, 54.0% of Tongans, 50.0% of Native Hawaiians, and 36.0% of Tahitians start college, but do not earn a college degree. These percentages show how the multicultural principle of attaining educational excellence is not manifesting for Pacific Island students. These percentages also display how challenges exist among marginalized groups of Pacific Island students (National Commission on Asian American Pacific Islander Research in Education, 2010). To reduce attrition rates, colleges need to recognize the needs and challenges of Pacific Island learners and start addressing how these needs and challenges contribute to low completion rates of Pacific Island students (National Commission on Asian American Pacific Islander Research in Education, 2010). For example, a considerable number Pacific Island students are English language learners (National Commission on Asian American Pacific Islander Research in Education, 2010). Because English is the dominant language in the United States, attending college in the United States is a challenge for many Pacific Island students (Phan, 2008).

Since English is the dominant language at the college under study, it follows that attending this college will be a challenge for many Pacific Island students; the statistics from the college suggest this is indeed the case. As reported by the college's Center for Academic Success, about 38% of freshman from the Pacific Islands obtain a GPA of less than 2.0 the first or second semester. Even though 38% is lower than the national average, there is ample room for improvement.

Striving for Excellence

ELLs who are not sufficiently proficient in English typically fall behind in their academic pursuits due to the additional load of learning material in a language they are struggling to understand (Abedi & Herman, 2010). To help ELLs succeed, academic institutions should offer English learners the opportunity to become familiar with academic language (Aguirre et al., 2006).

At the college where this study was completed, almost every classroom has a high proportion of foreign and English learners. To help with academic success, ELLs are required to take English support courses (for credit) during their freshman year. These courses are designed to help ELLs become more fluent in English while familiarizing them with general academic language. However, academic language in specific disciplines, such as mathematics, is not covered in these courses.

In addition to English support courses, the college employs full-time advisors. In other words, professors are not advisors. The responsibility of advisors at this college is to provide assistance, support, and guidance in every aspect of a student's academic life. Advisors also help students make realistic and applicable short and long term goals. Furthermore, the college has a

Center for Academic Success as well as areas for specialized tutoring. The college also employs reading and writing tutors who are specially certified to teach ELLs.

Purpose of the Study

Many professors mistakenly assume students' algebraic proficiency is adequate upon entering first semester calculus (Avila, 2013; Martin, 2000). Yet prior research literature indicates that students enter first semester calculus with weak algebra skills no matter students' prior mathematical achievements. However, prior research does not investigate the *degree* of algebraic weakness, in particular for ELL and foreign students. This said, the first purpose of this study is to determine just how weak ELL and foreign students' algebra skills are at the beginning and the end of first semester calculus. A second purpose is to compare the overall scores (in calculus) of ELLs with native English speakers. Bearing in mind the support offered to ELLs and foreign students by the college coupled with the use of functional language used by the calculus professors, a third purpose is to do a preliminary study on the effectiveness of the college's English preparation program and the efforts made by professors.

By determining the level of algebra proficiency for ELL and foreign students in calculus, more effort can be put forth into finding ways to increase student learning and decrease attrition rates. For example, Von Allmen (1996) found that when first-semester calculus was offered as a two-semester course which included a lot of algebra review, students were more successful in future economics courses. Today however, many departments do not have the luxury of requiring extra courses and additional credits for their majors. Therefore, alternative solutions must be considered.

One [immediate] solution is to help calculus instructors know how to quickly and efficiently measure the degree of their own students' algebra proficiency. By clearly and accurately knowing students' level of algebra proficiency, calculus instructors can modify their pedagogy to match students' algebra skills. By coordinating instruction with students' algebra proficiency, including the avoidance of skipping algebraic steps and using appropriate language, learning and understanding the calculus should increase and attrition rates should decrease.

To help instructors determine the degree of algebraic weakness in their own students, rapid assessment tests can be utilized (Kalyuga & Sweller, 2004). A rapid assessment test is a valid and efficient assessment instrument used to evaluate students' algebra proficiency. Rapid tests take very little class time to administer. For instance, in this study, students finished the rapid tests in less than seven minutes. Hence, a third purpose of this study to show how calculus instructors can use rapid assessment tests to measure their own students' level of algebra proficiency (Kalyuga & Sweller, 2004).

METHODOLOGY

Research Questions

The three main research questions addressed in this study are:

Question 1: How proficient are ELLs in algebra on day one of the course as compared to students whose native language is English?

Question 2: How proficient are ELLs in algebra at the end of the course as compared to students whose native language is English?

Question 3: Is there a difference in overall average scores (for calculus) between ELLs and native English speakers?

To answer the said questions a validated rapid assessment test, which measures algebra proficiency, was given on day one and at the end of the semester. A one sample proportion test, one sample t-test, and independent t-test were used to analyze the data. A two-sample *t*-test was utilized to compare the average score for calculus of ELLs with the average score of calculus for native English speakers.

The Sample and Its Limitations

The sample consisted of first semester calculus students, ages 18 – 25, from a small private liberal arts college in the western part of the United States during the spring 2016 and fall 2016 semesters. Approximately 50% of the student body at the said college are ELL or foreign students. Students enrolled in first semester calculus were used as the sample, making the sample a convenience sample. Since the sample was a convenience sample from a unique, small private liberal arts college, careful vigilance should be applied when extrapolating the results.

Data Sources and Instrumentation

Relative to algebra proficiency, data was collected at two distinct points in time throughout the two 13 week semesters where the 13th week was final exam week. Data was generated by giving a rapid assessment test created by Kalyuga and Sweller (2004). In the research done by Kalyuga and Sweller, rapid assessment tests were shown to be a viable measure of algebra proficiency. The test was duplicated and shown to be a viable measure of proficiency in coordinate geometry as well. The findings of Kalyuga and Sweller showed a significant correlation between students' achievement on the rapid tests and students' achievement on a more traditional test with a Pearson product-moment correlation coefficient of 0.92, $p < .01$. To match the conditions in Kalyuga and Sweller's study, students were urged to finish the rapid tests as quickly as possible to reveal immediate indications of content knowledge in students' memory and not outcomes of cognitive processes (Kalyuga & Sweller, 2004).

Each rapid assessment test contained 12 equations. The equations on the rapid tests are listed in Table 1. Rapid test 2 was exactly the same as rapid test 1. The first assessment test was administered on day one of the course. The second test took place during the last week of the semester. Each time a rapid test was administered, the data collected included the total number of correct answers for each student. Students were told the test scores on the rapid tests would in no way affect their grade in calculus; the rapid tests were strictly for research and were separate and independent from the calculus course. To avoid recall of questions on the first rapid test, students never saw their rapid test results and feedback was never given.

In addition to measuring algebra proficiency, the overall scores of ELLs were compared with the overall scores of native English speakers. The overall scores were determined by the scores on homework, three quizzes, four tests, and a final exam. The weights for the homework, quizzes, tests, and final exam, were respectively, 5%, 10%, 55%, and 30%.

Rapid Assessment Test # 1

Row Number			
1	$(x-1)(x+3)=2$	$x^2-2x+3=0$	$\tan^2\theta-2\tan\theta-24=0$
2	$9x^3-3x=0$	$x^3-3x^2+2x-6=0$	$x^4-13x^2+36=0$
3	$\frac{5}{x-1}=\frac{x+1}{x}$	$\frac{8}{x^2}+\frac{x}{2}=\frac{5}{x}$	$\frac{5}{x^{1/2}}=10x^{3/2}$
4	$\ln(x)+\ln(x+1)=0$	$e^{x+1}=12$	$(x-4)^{2/3}=9$

Table 1. Equations on Rapid Assessment Tests

How to Use Rapid Tests to Measure Algebra Proficiency

Rapid assessment tests can be given during class. To reveal immediate indications of content knowledge, time to complete each test should be limited to no more than seven or eight minutes. In other words, students should be urged to finish the rapid tests as quickly as possible. Rapid tests should contain 12 questions in four rows with three *similar* question types in each row. The 12 questions should be evenly spaced and presented on one page. Students are to write only the first step towards the solution. First step answers should be counted as correct or incorrect. No partial credit should be awarded. Thus, the highest possible score is 12. If a response is not an immediate first step answer but, is a latter step towards the solution, the answer should be counted as correct (Kalyuga & Sweller, 2004). For example, if the equation presented was $x^2-x-6=0$, an appropriate first step answer would be $(x-3)(x+2)=0$.

However, if the first step answer was $x-3=0, \quad x+2=0$ the answer should be counted as correct.

Hypotheses

Relative to the liberal arts college where the study was conducted, the hypotheses linked to algebra proficiency are:

1. The majority of ELL students entering first semester calculus have a rapid test score of 5 or lower AND this score is below the average score of American students whose native language is English.
2. HPCSA* students entering first semester calculus have an average rapid test score below 5 AND this score is below the average score of American students whose native language is English.
3. Asian students entering first semester calculus have an average rapid test score below 5 AND this score is equivalent to American students whose native language is English.
4. By the end of the semester the majority of ELL students have a rapid test score of 9 or lower AND this score is below the average score of American students whose native language is English.
5. By the end of the semester HPCSA* students have an average rapid test score below 9 AND this score is below the average score of American students whose native language is English.
6. By the end of the semester East Asian students have an average test score above 9 AND this score is equivalent to the average score of American students whose native language is English.

Algebraic proficiency was defined as a rapid test score of at least 9 (75%). For rapid test scores of 5 (41.7%) or lower, students were classified as extremely weak in algebra. Scores between 5 and 9 were classified as weak. To analyze hypotheses 1 and 4 a one sample proportion test was applied. To analyze hypotheses 2, 3, 5, and 6 a one sample t-test and independent t-test were utilized.

* **H**awaiian, **P**acific Island, **C**entral/**S**outh **A**merica

The hypothesis linked to the overall average scores was:

7. There is no difference between the overall average scores of ELLs and native English speakers.

RESULTS

Student Demographics and Withdrawals

The original sample consisted of 70 students but, 10 students withdrew. Of the 60 students who completed the course 18 were freshman, 23 were sophomores, 12 were juniors, and 7 were seniors. Twenty-nine were women and 31 were men. Seven students were mathematics or mathematics education majors, 47 were science majors and 6 were some other major. Twenty-nine (48.3%) were American whose native language was English, 26 (43.3%) were ELLs, 12 (20%) were from East Asia, and 19 (31.6%) were from Hawaii (Native Hawaiian), the Pacific Islands or Central/South America (HPCSA). In the latter group, 3 students were from Hawaii, 14 from the Pacific Islands, 1 from Central America, and 1 from South America. All 12 students from East Asia were ELLs while 14 of the 19 HPCSA students were ELLs (12 from the Pacific Islands, 1 from Central America, and 1 from South America).

Conditions for Inference

When using parametric tests to make inferences about the population under consideration, certain conditions need to be satisfied to insure valid results. Because one sample proportion tests, one sample t-tests, and independent t-tests are parametric tests and were used to analyze the data, their associated conditions are investigated.

One Sample Proportion Test

To ensure valid results when applying a one sample proportion test the following conditions need to be satisfied: $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$ where the null hypothesis takes the form $H_0 : p = \hat{p}$ (Gould & Ryan, 2013). For the 26 ELLs, $n\hat{p} = n(1-\hat{p}) = 26 \times 0.5 = 13 > 10$. Thus, the assumptions for a one sample proportion test are satisfied for the 26 ELLs. Since the null hypothesis for hypotheses 1 and 4 considers algebra proficiency of the majority of ELL students, the null hypothesis will be 50% are proficient, or $H_0 : p = .50$.

The t-Test

The assumptions for the proportion test are not satisfied for the small samples of 12 East Asian and 19 HPCSA students. Hence, a one sample t-test was considered. To warrant the use of a one sample t-test, the distribution in the population must be approximately normal or the sample size must be at least 25 (Gould & Ryan, 2013). The sample sizes of 12 and 19 are less than 25 thus, the stem & leaf and Q-Q plots were analyzed. For each of the two sample sizes the stem & leaf and the Q-Q plots showed distributions that were very close to normal.

An independent t-test was utilized to compare the average means between ELLs, HPCSA, East Asians, and Americans. The stem & leaf and Q-Q plots showed the Asian distribution was slightly skewed to the left and the American distribution was slightly skewed to the right. Nevertheless, both were sufficiently close to normal.

An independent two-sample t-test was employed to compare the overall average scores (in calculus) of ELLs with the overall average scores of native English speakers. The sample size for both samples were greater than 25. Further, the stem & leaf and Q-Q plots for both samples showed an approximately normal distribution.

Findings

Hypothesis 1

The first hypothesis was, the majority of ELL students entering first semester calculus have a rapid test score of 5 or lower AND this score is below the average score of American students whose native language is English. To determine if Hypothesis 1 is statistically significant, a one sample proportion test was run on the ELL scores for rapid test 1. The hypotheses were:

H_0: The proportion of ELL students who scored 5 or less on test 1 was less than 0.5

H_a: The proportion of students who scored 5 or less on test 1 was greater than 0.5

Formally written, the hypotheses were:

$$H_0: \text{p} = 0.5$$
$$H_a: \text{p} > 0.5$$

The proportion of ELLs who scored 5 or lower was 20/26 (77%) with an average test score of 4.31 (36%), p = .003. The median score was 4. The results indicate that the majority of ELL students are extremely weak in algebra.

Comparatively, the average rapid test score for American students, whose native language was English, was 3.21 and the median score was 3. Because the average test score for ELLs is larger than the average for Americans, an independent two sample t-test was run using the following hypotheses:

$$H_0: \mu_{ELL} - \mu_{Am} = 0$$
$$H_a: \mu_{ELL} - \mu_{Am} > 0$$

The data produced a p-value of .0152, therefore H_0 is rejected and the evidence suggests that ELL's algebra proficiency is statistically significantly higher than American students on day one of calculus.

Hypothesis 2

Hypothesis 2 was, HPCSA students entering first semester calculus have an average rapid test score below 5 AND this score is below the average score of American students whose native language is English. The hypotheses for the one sample t-test were:

$$H_0 : \mu = 5$$
$$H_a : \mu < 5$$

Of the 19 HPCSA students, the average score for rapid test 1 was 3.11 (26%), $p < .0001$. The results indicate that the average test score for HPCSA students was statistically significantly less than 5 and thus, HPCSA students are extremely weak in algebra on day one of calculus.

To compare the average scores of HPCSA students with the average score of American students, the hypotheses for the independent t-test were:

$$H_0 : \mu_{HPCSA} - \mu_{Am} = 0$$
$$H_a : \mu_{HPCSA} - \mu_{Am} < 0$$

The data produced a p-value of .4157 suggesting that HPCSA's algebra proficiency is equivalent to American students on day one of calculus.

Hypothesis 3

Hypothesis 3 was, Asian students entering first semester calculus have an average rapid test score below 5 AND this score is equivalent to American students whose native language is English. The hypotheses for the one sample t-test were:

$$H_0 : \mu = 5$$
$$H_a : \mu < 5$$

Of the 12 Asian students, the average score was 5.00 (41.7%), $p = .5$. The results suggest that the average score is 5. Thus, Asian students are extremely weak in algebra on day one of calculus.

The hypotheses for the independent t-test were:

$$H_0 : \mu_{Asian} - \mu_{Am} = 0$$
$$H_a : \mu_{Asian} - \mu_{Am} > 0$$

The average rapid test score for American students whose native language was English was 3.21. The independent t-test had a p-value of .0098. The results suggest that Asian students had a statistically significantly higher average score than American students on day one of calculus.

Hypothesis 4

The fourth hypothesis was, by the end of the semester the majority of ELL students have a rapid test score of 9 or lower AND this score is below the average score of American students whose native language is English.

The hypotheses for the proportion test were:

$$H_0 : \text{p} = 0.5$$
$$H_a : \text{p} > 0.5$$

The proportion of ELL students who scored less than 9 was 12/26 (46%) with an average test score of 8.23 (69%), $p = .6526$. The results indicate that the majority of ELL students are proficient in algebra at the end of the course. To supplement these results, a one sample t-test was run on the hypotheses:

$$H_0 : \mu = 9$$
$$H_a : \mu < 9$$

The p-value was .0696 suggesting the average score for ELLs was 9. Combining the results of the proportion test with those of the one sample t-test, the evidence suggests ELLs have an average score of 9 and are thus proficient in algebra at the end of calculus.

The hypotheses for the independent t-test were:

$$H_0 : \mu_{ELL} - \mu_{Am} = 0$$
$$H_a : \mu_{ELL} - \mu_{Am} > 0$$

The average score for the American students on rapid test 2 was 7.89. The independent t-test had a p-value of .2886. The results indicate that ELLs and American students whose native language was English had equivalent levels of algebra proficiency at the end of the semester.

Hypothesis 5

Hypothesis 5 was, by the end of the semester HPCSA students have an average rapid test score below 9 AND this score is below the average score of American students whose native language is English. The hypotheses for the one sample t-test were:

$$H_0 : \mu = 9$$
$$H_a : \mu < 9$$

Of the 19 HPCSA students, the average test score was 6.16 (51%), $p < .0001$. The results indicate that the average score on test 2 for HPCSA students was statistically significantly less than 9. Thus, HPCSA students are weak in algebra at the end of calculus.

The hypotheses for the independent t-test were:

$$H_0 : \mu_{HPCSA} - \mu_{Am} = 0$$
$$H_a : \mu_{HPCSA} - \mu_{Am} < 0$$

The independent t-test had a p-value of .0063. The results suggest that at the end of the semester, HPICSA students had an average score that was statistically significantly less than American students.

Hypothesis 6

Hypothesis 6 was, by the end of the semester East Asian students have an average test score above 9 AND this score is equivalent to the average score of American students whose native language is English. The hypotheses for the one sample t-test were:

$$H_0 : \mu = 9$$
$$H_a : \mu > 9$$

Of the 12 East Asia students, the average test score was 9.67 (81%), p = .1893. The results indicate that the average score on test 2 for East Asia students is 9. Thus, East Asia students are proficient in algebra at the end of calculus.

The hypotheses for the independent t-test were:

$$H_0 : \mu_{Asia} - \mu_{Am} = 0$$
$$H_a : \mu_{Asia} - \mu_{Am} > 0$$

The independent t-test had a p-value of .0214. The results suggest that at the end of the semester, East Asia students had an average score that was statistically significantly greater than American students.

Hypothesis 7

Hypothesis 7 was, there is no difference between the overall average scores of ELLs and native English speakers. The hypotheses for the two-sample t-test were:

$$H_0 : \mu_{ELL} - \mu_{ENG} = 0$$
$$H_a : \mu_{ELL} - \mu_{ENG} \neq 0$$

The ELLs had an average overall score of 82.0 and the native English speakers had an average overall score of 77.2. The p-value was .1450. The results indicate that at the end of the semester there was no significant difference between the overall average scores of ELLs and native English speakers.

DISCUSSION

Proficiency in both algebra and calculus were studied in the following four groups of students enrolled in first semester calculus: (1) ELLs, (2) HPCSA, (3) East Asian, and (4) American. Upon entering calculus, all four groups were found to be *extremely* weak in algebra. The average scores (out of 12) for rapid test 1 were HPCSA (3.11), American (3.21), ELL (4.31), and Asian (5.00). The average scores for rapid test 2 were HPCSA (6.16), American (7.89), ELL (8.23), and Asian (9.67). Of the four groups, HPCSA students performed the poorest with an average score of 3.11 (26%) and 6.16 (51%) on rapid tests 1 and 2 respectively. Asian students performed the best with an average score of 5.00 (42%) and 9.67 (81%) on rapid tests 1 and 2 respectively. Relative to overall performance in calculus, there was not a significant difference between ELLs and native English speakers. However, it is worth pointing out that the overall average in calculus of ELLs was 82.0 while the overall average of native English speakers was 77.2

A one sample t-test with null and alternative hypotheses $H_0 : \mu = 9$ and $H_a : \mu < 9$ respectively showed that by the end of the semester, the average rapid test score for all ELLs was 9 (or 75%). Therefore, collectively, the ELLs were classified as proficient in algebra at the end of the semester. The subgroup of 12 Asian ELLs were also found to be proficient in algebra. Altogether, ELLs scored statistically significantly higher than American students (whose native language was English) on both tests. It should be noted however, about half (46%) of the ELL group were Asian, and Asians tend to perform better in mathematics than other sectors of the foreign population (Barrett, Barile, Malm & Weaver, 2012). Future research addressing why ELLs outperformed native English speakers will be carried out at the college where this study was conducted. Specifically, data will be collected as part of a longitudinal study. Overall achievement levels for marginalized groups will also be investigated.

In addition to ELLs outperforming Americans, HPCSA's algebra proficiency was found to be statistically equivalent to American students on day one of calculus but, by the end of the semester, HPCSA's algebra proficiency was statistically significantly less than American students. Relative to Americans, the smaller increase for HPCSAs could be due to a number of reasons. One, English was the native language for 5 of the 19 (26%) HPCSA students so the English preparation programs might show less of a global effect on this group. Two, because HPCSA students are marginalized, oppression is another confounding variable (National Commission on Asian American Pacific Islander Research in Education, 2010).

As discussed in the Striving for Excellence section in the Introduction, the college in this study puts forth significant effort to prepare and sustain foreign and ELL students for academic success. To help sustain these students, the professors who taught the calculus courses paid special attention to functional language while teaching, including academic language. Though cause and effect cannot be determined from this study, it is the hope that such efforts played a role in the success of ELLs in first semester calculus. As mentioned earlier, after more data is collected over a longer period of time, future research will investigate the effects the college's efforts have on the academic success of ELLs.

Clear Explanations

Because the college under study has a high proportion of ELL and foreign students, faculty are trained and continually reminded about their use of English while teaching. The professors who taught the calculus courses paid special attention to the functional use of English while teaching. To foster mathematical success while improving proficiency in English, when appropriate, effort was made to clearly define mathematical terms while using graphics (Aguirre et al., 2006). Graphics were also used to clarify "common" English terminology such as shifting, perimeter, scaling, and even more basic words like size (Borgioli, 2008). The professors also paid special attention when writing tests, trying to phrase questions as clearly and understandably as possible. Nevertheless, there is plenty of room for improvement.

During the last test of the semester (a test that counted towards student's grade, not the rapid test), several ELLs asked for linguistic clarification. For example, one test question asked students to find the area under a curve by using geometry. Some students did not understand what "using geometry" meant. Similarly, for another problem that said to use Riemann sums to estimate the area, ELLs had questions about what this meant even though the terminology "Riemann sums" was used in several classes and the homework.

Another question that generated confusion during the last test of the semester was, "Find the area bounded above by $y = x^2 + 2$ and below by $y = 9x^2$." Seven of the 12 (58%) Asian students were puzzled by the words "above" and "below." In addition, even though these students did not specifically ask about the word "bounded", it became evident that bounded was misunderstood as well because all 7 students thought the area to be calculated was the unbounded region between the two curves. Through informal conversations, the professors realized these students would have been less puzzled if the question was phrased as "Find the area bounded by $y = x^2 + 2$ and $y = 9x^2$." The interesting part is, even though "bounded" was not understood, it was the words "above" and "below" that baffled the students. Furthermore, it was only Asian students who asked for language clarification. Not one American or HPCSA asked for interpretative help on this question. While grading the tests, it was clear the American and HPCSA students understood the wording because the bounded area was properly shaded on their graph. It should also be noted that the “using geometry” and the “area between curves” problems were the only problems students asked for interpretive help.

How to write understandable and equitable mathematics tests through effective use of language is a question that has yet to be clearly answered (Sato, Rabinowitz, Gallagher & Huang, 2010). According to Sato et al., the effectiveness of language modification on math tests varies between ELL groups and non-ELL groups. For the question “Find the area bounded above by $y = x^2 + 2$ and below by $y = 9x^2$”, only Asian students asked for assistance with interpretation, which supports Sato’s supposition.

Skipping Steps

In addition to putting forth their best effort with written language, spoken language, and graphics, the professors who taught the courses avoided skipping algebraic steps especially during the first half of the semester when students were exceptionally weak in algebra. Towards the end of the course algebraic steps were slowly and gradually omitted. However, every time steps were skipped, verbal explanations were included and students were asked if they understood. If a student said they were unable to follow, the algebraic procedure was written without skipping steps. The point is, mathematics is a language and when a person is learning a language, if it is spoken or written too quickly, the language learner will struggle to comprehend what is spoken or written (Chand, 2007). Therefore, if mathematical language is spoken or written too quickly, or parts are omitted altogether (e.g. skipping steps), students who are not mathematically fluent will become lost. Because both ELL and non-ELL calculus students are extremely weak in algebra at the beginning of calculus, it follows that skipping algebraic steps should be avoided towards the beginning of the semester.

Rapid Assessment Tests

Quality instruction in mathematics courses is positively correlated with student learning (Carrell & West, 2008). Research suggests that quality instruction could be the strongest contributing variable to teaching ELLs (Calderón, Slavin & Sánchez, 2011). Further, knowing students by consistently utilizing assessment tools is an important resource for effective instruction of mathematics (Ball, 1997; Even & Tirosh, 2002; Kuh, Jankowski, Ikenberry & Kinzie, 2014). The use of rapid assessment tests can help instructors gauge the level of algebra proficiency in their own students. By knowing the level of students' proficiency, professors can match instructional techniques with the skill level of the students (Kalyuga & Sweller, 2004). If instructors skip algebraic steps while ELL students lack the ability to do mental algebra, students' working memory will become overloaded. Professors are therefore encouraged to utilize rapid assessment tests in their classroom.

Future Research

As discussed earlier, additional research will be carried out to analyze the effects the English preparation programs have on ELLs. Because pace is a contributing factor to learning, included in that research will be the pace at which information is delivered during class (Pan & Tang, 2005). According to Goldenberg (2008), the pace at which information is delivered in the classroom is critical for effective learning. In the English-only calculus classroom, content is taught and learned using English. Hence, the pace at which mathematical content is verbally delivered is directly proportional with the pace at which English is used. Thus, to increase access and equitability for ELLs, instructors must be cognizant of the pace at which they deliver content.

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