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TEACHER PERSPECTIVES ON THE EDUCATIONAL IMPACT OF RESEARCH EXPERIENCES IN MATHEMATICS FOR UNDERGRADUATES AND TEACHERS CALIFORNIA STATE UNIVERSITY, CHICO

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Synopsis:

Peter Gerrodette and Dan Sours, two teacher participants in the California State University, Chico Research Experiences in Mathematics for Undergraduates and Teachers share how their participation changed them as educators and informed their classroom pedagogy.

Teacher Perspectives on the Educational Impact of Research Experiences in Mathematics for Undergraduates and Teachers California State University, Chico.

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Abstract

Research Experiences for Undergraduates (REU) have existed for almost 60 years. The REU programs are competitive summer research programs for undergraduates studying science, engineering, or mathematics. The programs are sponsored by the National Science Foundation, and are hosted in various universities. REUs tend to be specialized in a particular field of science. There have been over 60 REUs in mathematics all over the United States for most of that 60 years. Undergraduate students (normally juniors or seniors) apply to be part of a research team that will investigate a particular mathematical topic. By its very nature, these topics are not the in the areas normally studied by undergraduates or at the very least, major extensions of standard topics. The history of REUs is important to us because neither of us were aware of the program's existence while we were undergraduates in the 70's and 80's. In fact, the concept that research in mathematics was even accessible to anyone not referenced in a textbook was a foreign concept. We knew that people like Andrew Wiles at Oxford were working on proving things like "Fermat's Last Theorem" but it was never a consideration that we (or anyone we know) could go to CSU, San Bernardino and prove what happens when you cut up a mobius strip.

In 2002 California State University, Chico decided that adding inservice high school math teachers would benefit all involved. We feel that both teachers and undergraduates benefit from working together on a research team. Participants develop connections not only with faculty but also with participants who have very different backgrounds, experiences, and career paths. Students have the opportunity to develop a direct relationship with a math educator; they benefit from the experience teachers have in communicating mathematical ideas and may be inspired to consider the possibility of teaching as a career. Secondary teachers also benefit from the experience of close contact with outstanding undergraduates and see firsthand what some of their current students will be doing in a few years. Both populations benefit from their complementary mathematical backgrounds, which allow them to help one another in their research exploration. A teacher's broader mathematical experience can provide the insight to make a conjecture, while a student's recent exposure to college-level mathematics could provide the specific tools to "make the epsilons and deltas work." We provide a research experience that enhances the skills necessary for careers in any of the sciences, while adding both depth and breadth to the participant's mathematical knowledge, skill, and understanding. While seeing the value of including inservice teachers, the National Science Foundation was not convinced that

enough teachers would be interested in participating to maintain the volume necessary to insure a quality program. As a regular participant in CSUC Math Department programs, Dan Sours was approached by professors at California State University, Chico and asked if he believed that teachers would support the idea of becoming involved in the Research Experience for Undergraduates program. Dan was very excited about the concept. This was at least partly because, as described earlier, the existence of the REU program was never brought to his attention when he was an undergraduate so math research was not a part of his education and as a teacher of mathematics, he thoroughly enjoyed the problem solving process and loved the opportunity to do challenging mathematics. He especially appreciated the idea of working on problems that had not been fully explored, as this is not the kind of mathematics done as a regular part of his teaching. However, most exciting was the opportunity to do interesting mathematics with motivated intelligent people, this was irresistible. All of those reasons seem to have resonated with his colleagues as well, as each year of the REUT in Chico has been very competitive with far more teachers applying than there has been room to accommodate. Dan Sours has done the REUT three times, each time working in a different area. Starting in Knot Theory he defined and gave a process for classification of paradromic rings as either torus links or torus links with an alternating core and then described the colourability of those links. Then in Mathematical Modeling, Mathematica was used to investigate the steady and non-steady flows of a liquid polymer treated as a non-Newtonian fluid on the inner surface of a horizontal rotating cylinder are investigated, the governing equations for non-steady Power-Law and Ellis fluids are solved numerically and the time of transition from non-steady to steady-state mode for various model parameters and flow conditions were defined and by extension, the stabilization effect of a chemical reaction within the polymeric fluid (reactive flow) was examined. Lastly, in Statistics, confidence intervals were compared with Bayesian credible intervals under a variety of scenarios to determine when Bayesian credible intervals outperform frequentist confidence intervals. Results indicated that Bayesian interval estimation frequently produces results with precision greater than or equal to the frequentist method. Prior to the beginning of each program, Dan knew little or nothing about any of these topics, yet each time he became an integral part of a community that made each topic that much better understood. Peter Gerrodette on the other hand has attended the Chico REUT quite recently in 2014 and was involved in a tsunami modeling project. Peter Gerrodette and Dan Sours, two teacher participants in the California State University, Chico Research Experiences in Mathematics for Undergraduates and Teachers share how their participation changed them as educators and informed their classroom pedagogy.

Introduction

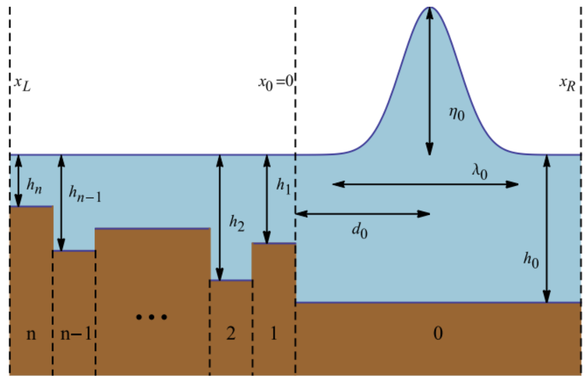
In 2002 California State University, Chico decided to apply to NSF for REU grant. In that application it was decided that adding inservice high school math teachers to the program would benefit all involved. We feel that both teachers and undergraduates benefit from working together on a research team. Participants develop connections not only with faculty but also with participants who have very different backgrounds, experiences, and career paths. Students have the opportunity to develop a direct relationship with a math educator; they benefit from the experience teachers have in communicating mathematical ideas and may be inspired to consider the possibility of teaching as a career. Secondary teachers also benefit from the experience of close contact with outstanding undergraduates and see firsthand what some of their current students will be doing in a few years. Both populations benefit from their complementary mathematical backgrounds, which allow them to help one another in their research exploration. A teacher's broader mathematical experience can provide the insight to make a conjecture, while a student's recent exposure to college-level mathematics could provide the specific tools to "make the epsilons and deltas work." We provide a research experience that enhances the skills necessary for careers in any of the sciences, while adding both depth and breadth to the participant's mathematical knowledge, skill, and understanding [1].

While seeing the value of including inservice teachers, the National Science Foundation was not convinced that enough teachers would be interested in participating to maintain the volume necessary to insure a quality program. Below, Peter Gerrodette and Dan Sours, two teacher participants in the California State University, Chico Research Experiences in Mathematics for Undergraduates and Teachers share some of the research problems they helped investigate and how their participation changed them as educators and informs their classroom pedagogy.

(Dan Sours): As a regular participant in CSUC Math Department programs, I was approached by professors at California State University, Chico and asked if I believed that teachers would support the idea of becoming involved in the Research Experience for Undergraduates program. I was very excited about the concept. This was at least partly because, as described earlier, the existence of the REU program was never brought to my attention when I was an undergraduate, so math research was not a part of my education. The "newness" excited me. As a teacher of mathematics, I thoroughly enjoy the problem solving process and love the opportunity to do challenging mathematics. I especially appreciated the idea of working on problems that had not been fully explored, as this is not the kind of mathematics done as a regular part of my teaching. However, most exciting was the opportunity to do interesting mathematics with motivated intelligent people, this was irresistible. All of those reasons seem to have resonated with my colleagues as well, as each year of the REUT in Chico has been very competitive with far more teachers applying than there has been room to accommodate.

(Dan): I have done the REUT four times. Starting in Knot Theory I helped define and give a process for classification of paradromic rings as either torus links or torus links with an alternating core and then described the colourability of those links. Then in Mathematical Modeling, Mathematica was used to investigate the steady and non-steady flows of a liquid polymer treated as a non-Newtonian fluid on the inner surface of a horizontal rotating cylinder are investigated, the governing equations for non-steady Power-Law and Ellis fluids are solved numerically and the time of transition from non-steady to steady-state mode for various model parameters and flow conditions were defined and by extension, the stabilization effect of a chemical reaction within the polymeric fluid (reactive flow) was examined. Next, in Statistics, confidence intervals were compared with Bayesian credible intervals under a variety of scenarios to determine when Bayesian credible intervals outperform frequentist confidence intervals. Results indicated that Bayesian interval estimation frequently produces results with precision greater than or equal to the frequentist method. Currently, I am working with a statistics group to use simulations run in the R programming language that will determine the optimum choice for NFL teams "going for it" on fourth down.

(Peter): I have done the REUT twice now with the Applied Mathematics group. Both sessions involved mathematically modeling, numerically and analytically, tsunami waves as they pass over a variety of sea-floor configurations such as an underwater shelf, and underwater shelf with a sloped beach, a flat sea-floor with a single obstacle, and a series of obstacles. (See figure at right)



Prior to the beginning of the program, both Dan and Peter knew little or nothing about any of these topics, yet each time they became an integral part of a community that made each topic that much better understood.

California State University, Chico (CSUC) has been a teachers college since its inception. The goals of the REUT mesh closely with CSUC's mission statement, and hence there is strong institutional support for such a program. In particular, teaching is the institution's primary mission and CSUC embraces a teacher/scholar model in which the main purpose of research is to improve the undergraduate experience. Another guiding mission for CSUC is teacher preparation and support of the K-12 community. Many high school mathematics teachers in CSUC's large rural service area already have strong ties to the department and college. The majority of the area's teachers received their bachelor's degree or teaching credential at CSUC. The university is also a home to the Chico Mathematics Project (CMP), which is charged by the State with providing professional development for in-service mathematics teachers at all grade levels. In addition, CSUC has a Master's program in mathematics education, populated entirely by in-service mathematics teachers who attend the program during three consecutive summers. Several of the fifteen teachers funded through the previous REUT grants have connections with these programs. In fact, three of the fifteen completed Master's theses based on their REUT research and a five are alumnus of the Master's program. Thus, CSUC's REUT fits strongly within the teacher-scholar model and complements and extends the broad range of mathematical activities occurring in Chico during the summer.

Both Dan and Peter fall into all of those categories. Both received our undergraduate degree and teaching credential and Dan his master's degree from Chico State and both past multiple time participants in the California Math Project. Most important in their decision to be a part of the REUT, is a keen interest in expanding our knowledge of mathematics and mathematics education. The opportunity to do interesting mathematics with motivated intelligent people was irresistible.

Research experience gained by teachers during the summer REUT program

(Peter) My initial reaction as I walked into Room185 of Holt Hall and looked at the board which was filled with equations that the undergraduates had already been working on for a week was, "Oh my! What have I gotten myself into?" Typically, when I look at equations, they mean something to me. Analogous to hearing the punchline without hearing the joke, with these equations (see below: 78, 81,82),

$$\eta_0(y,t) = \sum_{k=1}^{\infty} \left[\hat{f}(\lambda_k)\cos(\lambda_k t)\frac{\sqrt{2}}{\sqrt{L}} \frac{J_0\left(\sqrt{\frac{4\lambda_k^2 y}{\alpha}}\right)}{\left|J_1\left(\sqrt{\frac{4\lambda_k^2 L}{\alpha}}\right)\right|} \right] \qquad (78)$$

for some arbitrary function $\psi(y)$. However, the initial condition $u_0(y,0) = 0$ gives us that $\psi(y) = 0$, and so

$$u_0(y,t) = \sum_{k=1}^{\infty} \left[\hat{f}(\lambda_k)\sin(\lambda_k t)\sqrt{\frac{2}{\alpha L y}} \frac{J_1\left(\sqrt{\frac{4\lambda_k^2 y}{\alpha}}\right)}{\left|J_1\left(\sqrt{\frac{4\lambda_k^2 L}{\alpha}}\right)\right|} \right] \qquad (81)$$

Finally, since we know $\eta_0(0,t) + \alpha x_0 = 0$, and $J_0(0) = 1$, we see that

$$x_0(t) = \sum_{k=1}^{\infty} \left[\frac{\hat{f}(\lambda_k)\sqrt{2}\cos(\lambda_k t)}{-\alpha\sqrt{L}\left|J_1\left(\sqrt{\frac{4\lambda_k^2 L}{\alpha}}\right)\right|} \right] \qquad (82)$$

I felt out of my depth and a little uncomfortable. Maybe it would be helpful to define some variables. $\eta_0(y,t)$ represents the first term of the approximation of η, the amplitude of the wave, $u_0(y,t)$ represents the fluid velocity with respect to space and time and $x_0(t)$ represents the position of the initial wave profile with respect to time. It has been nearly thirty years since I studied partial differential equations (pde's) as an undergrad. At that point in my math history, I considered myself to be a good math student; however, I was very good at memorizing definitions, algorithms and techniques; honestly, I have to admit that I probably didn't understand pde's concretely. Most would forgive me. After all, aren't equations by their very nature abstract objects and must necessarily be inaccessible? Now that I teach mathematics, I believe that real understanding means understanding conceptually and when I see equations like the Pythagorean Theorem, the Law of Cosines or the Fundamental Theorem of Calculus, my understanding is concrete - I have many connections and representations that I can use to communicate meaning. Less so for equations like those above but as I study them more, my connections are increasing and my understanding is becoming less abstract and more concrete.

Wilensky [8] proposes that “concreteness is not a property of an object but rather a property of a person's relationship to an object. Concepts that were hopelessly abstract at one time can become concrete for us if we get into the "right relationship" with them. The more connections we make between an object and other objects, the more concrete it becomes for us. The richer the set of representations of the object, the more ways we have of interacting with it, the more concrete it is for us. Concreteness, then, is that property which measures the degree of our relatedness to the object, (the richness of our representations, interactions, connections with the object), how close we are to it, or, if you will, the quality of our relationship with the object.”

It has been said that content area knowledge is one of the core key element in determining effective teaching. If you have a keen interest in taking your mathematical content knowledge to the limit there is no better resource than the REUT. You will quickly discover that mathematics is exactly not like riding a bike. In the REUT you will be immediately be re-introduced (or in most cases introduced) to Mathematics that you have not seen in many years. As an example, computer modeling was in its infancy when we were undergraduates and now writing code in Mathematica or R is often a major element in mathematical research. Even a problem that seems so hands on like twisting strips of paper, attaching the ends and cutting them up to make paradromic rings quickly evolves into finding the determinants of nxn matrices using Mathematica. Since the advent of relatively inexpensive computing power, virtually all statistically analysis is done with simulations in code in a programming language like R. Even though mathematical modeling of the action of flows of a liquid polymer treated as a non-Newtonian fluid on the inner surface of a horizontal rotating cylinder or the effect underwater obstacles have on a tsunami can be addressed by finding the solutions to differential equations you don’t have a clue how to solve, at a minimum those results need to be checked using code in Mathematica. More often the actual results are found using that code. What we are saying is that you should know what kind of a mindset you have, or perhaps the kind of mindset you would like to develop, before you decide to become the T in REUT.

According to the self-theories proposed by Dweck [1], individuals hold beliefs about the nature of their personal traits, referred to as their mindset, which can be classified into one of two core beliefs. Those with the fixed mindset believe their traits are an entity that cannot be changed. They tend to reduce their level of practice when they encounter difficulty. Conversely, those with the growth mindset believe their traits are flexible and can be enhanced through effort. They tend to maintain regular practice despite the level of difficulty.

If you have a fixed mindset you will immediately be intimidated by the undergraduates that you will be working with. Their mathematical knowledge base is much more extensive and current than ours which has atrophied over the decades. They will have specialized in the area of mathematics you will be working with more recently than you and unless you live and breathe coding, they will be able to type code faster than you can type. You will not be the smartest person in the room. If that is intimidating to you, the REUT is not for you. If you see mistakes and failures as a necessary part of learning mathematics, the REUT is for you. If you are comfortable with uncertainty about how to solve a problem, the REUT is for you. So if you are no longer the expert authority in the room, what specifically would be your role? Your broader mathematical experience can provide the insights necessary for making a workable conjecture. From an intellectual standpoint, we like to say, “we are like the village idiot.” That is, we are the guy that can ask the questions that help move the research forward and provide the prompts that cause the undergraduates to reflect on their thinking (e.g. How do you know that? Can you explain to me what that result means? Explain how the nonlinear portion of the partial differential equation is reflected the behavior of the wave. Explain what that piece of code does. Why do you think the code isn’t doing what you want it to do?) Good questioning strategies not only help move the research forward, but also create opportunities to explain to us their thinking which helps them formulate their research in a way that will make it clearer when they write up their results. Students will benefit from your experienced ability to communicate mathematical ideas. In addition, participants have very different backgrounds, experiences, and career paths, and you will often be in the position of making sure those differences are seen as complementary and ensure that everyone is working together in their research exploration. If you have a growth mindset and are comfortable with taking on the role of guide instead of sage, you will be an integral part of facilitating the research.

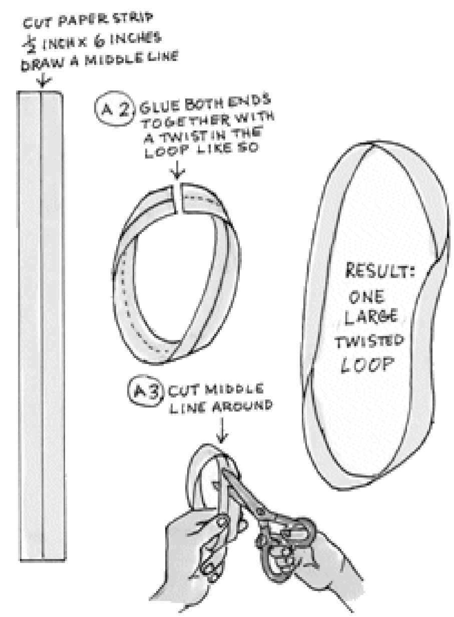
The abstract for the National Science Foundation grant at California State University, Chico states in part [3]: The research experience is intended to give participants an appreciation for the breadth and depth of mathematics and its applications, while providing undergraduates an opportunity to improve their communication skills and in-service teachers an experience that will deepen their understanding of mathematical content and inspire pedagogical innovation. By working on open problems in mathematics, participants will experience the excitement of exploration, discovery, analysis, proof, and systematization that are part of the mathematician's world. While much of mathematics is accessible only after years of study, the field is rich enough to allow for a full mathematical experience at the undergraduate level.

As the T in REUT you are in a unique position, you are not the expert authority in the area you will be studying. You will have a broader mathematical experience that allows you to have greater insight into making a conjecture. Your teaching experience provides a repository of key guiding questions to ask when immersed in the uncertainty of a problem.

(Dan) I was working with a group that was trying to solve a probability problem using code written in R and we were stuck trying to figure out what they needed to do to model the problem. I was able to say “Well, if I was having students in my class do this problem, I would have them roll a die, if the same number came up twice, I would have them roll again and ignore the roll that was the same.” That was enough for them to start writing the code and that would answer the question.

(Peter): I was working with one group trying to model the ocean-floor topography, replacing the shelves and bumps composed of linear segments that contained non-differentiable “corners”, with smoothening functions that would be differentiable At first they utilized an *arctan* function, but I proposed that a cubic may work better for curve-fitting. They were uncertain about exactly how to find the coefficients for the general cubic equation. I asked, "What do we know?" They responded with the four boundary conditions: that the derivatives on either side of the "corner" equal zero, that the height (depth) of the ocean floor equals one and that the height of the shelf is (1- k_2), which was sufficient in order to generate a system of four equations and four unknowns, which we then solved, to find the coefficients for the general cubic.

(Dan): Topologists have been described as those people who cannot distinguish a coffee cup from a doughnut. (They are both examples of a torus.) However, for me, it is the Mobius strip that is the surface most evocative of topology. History is on my side: the man who invented the word "topology," Johann Listing, is also credited with co-discovering the Mobius strip. But the real reason is that I will never forget the experience of being challenged by someone older and wiser to color one side of a Mobius strip red and the other blue. The second exercise in experimental topology, bisecting a Mobius strip, is per-haps even more memorable. In fact, the result is so surprising that I insist that if you have never seen this demonstration put aside this reading, gather up some paper, tape, and scissors, and see for yourself what happens when a Mobius strip is cut in half lengthwise (see figure below).



The results are so interesting that one of my current textbooks has students do this activity to show them how to make predictions and test hypotheses. In fact, it was a student's reaction to this exercise that caused me to want to investigate paradromic rings. Natural extensions include halving the resulting construction again or else starting over and cutting the strip in thirds, or, in general, into n sections. Or instead of joining the ends of the paper strip with a single half twist, make two twists, or three, or, in general, m twists. The various collections of loops generated by shredding a strip of paper in this way are called paradromic rings and will be denoted *P(m, n).*

In the paper that was written, we combined those two famous experiments by investigating the colorability of the paradromic rings. First, we were able to classify the paradromic rings using knot theory and see that they are closely related to a well understood family of knots, the torus knots. Then, using the idea of *p*-colorability, we determined which prime "colors" p belong to each *P(m, n).* Remarkably, almost all paradromic rings arc either invisible and take no colors or else rainbow rings and have a p- coloring for absolutely every prime *p*.[6]

(Peter): Let me attempt to provide a synopsis of our tsunami research during the 2014 and 2016 REUTs. I'll begin with a snippet from the abstract of one of the research papers produced during the 2014 REU. "A simple and easily implementable numerical scheme using explicit finite difference methods to solve the discontinuous partial differential equations is developed. The numerical solutions were verified with the exact analytical solutions of linear wave propagation over an underwater shelf. The scope of this simplified approach is determined by comparison of its results to those of another numerical solution and wave transmission and reflection coefficients from experimental data available in the literature. The efficacy of approximating more complicated continuous underwater topographies by piecewise constant distributions is determined. As an application, a series of underwater obstacles is implemented [2]." When the authors use the word "simple", it's evident that their understanding of the problem is very concrete, and as I read this, it does make more sense to me now. At the time, however, I felt like I was in over my head and couldn't touch the bottom.

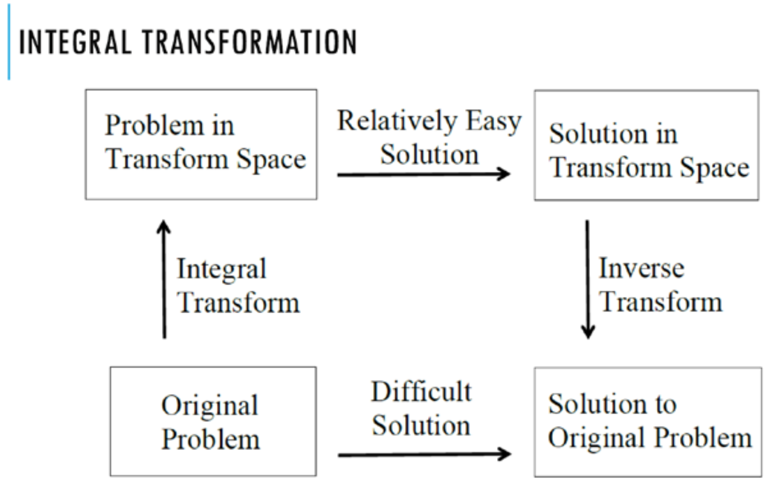
In the Applied Math REU's, all the groups started with the nonlinear shallow-water equations (Stoker,1957; Voltsinger et al., 1989; Whitham, 1974).

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + g\frac{\partial \eta'}{\partial x'} = 0 \qquad (1)$$

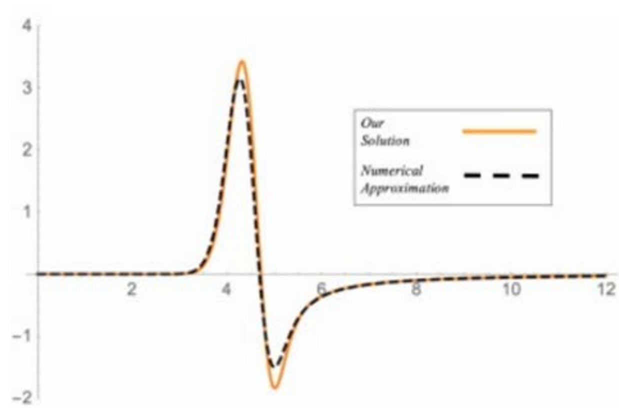
$$\frac{\partial \eta'}{\partial t'} + \frac{\partial}{\partial x}[(\eta' + \varphi')u'] = 0 \qquad (2)$$

And boundary conditions which are contingent upon the ocean-floor topography (eg. shelf, bump (or multiple bumps), or sloping beach run-up and run-down). Changing the boundary conditions changes the solution. Note that equations 78, 81 and 82 from above are in fact the approximate analytical solution for the sloping beach problem. I'll try to describe how they solved it without showing the eighty or so steps of the derivation. (See figure below)

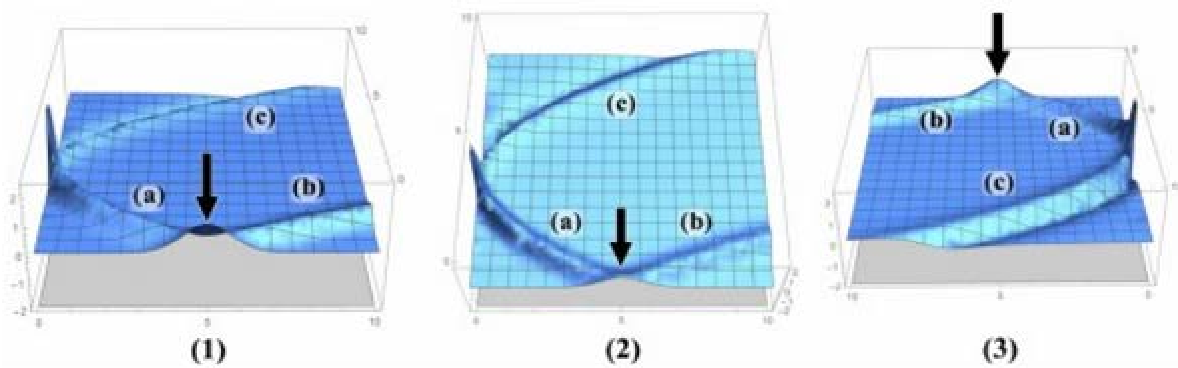
First, they made substitutions in order to nondimensionalize the system of equations; in other words, remove the units (eg. meter/sec, feet etc.). Second they made more substitutions to eliminate the moving boundary as the position of the wave moves up and down the beach. Next, a Taylor Series approximation (about y) is applied to simplify $\varphi(y - \varepsilon x_m)$, then they used McClaurin expansions for η and φ, and applied the Method of Perturbations to unknown functions u and x_m. Finally to solve their equations they used a method of integral transformations. (see figure below)



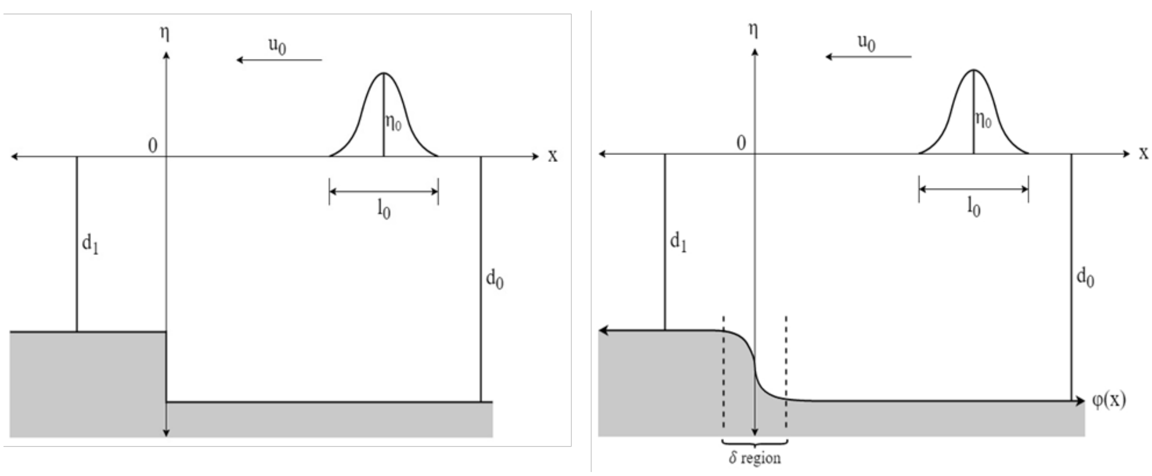
To validate their approximate analytical solution, they use Mathematica to compare with the numerical solution. When modeling with mathematics (Math Practice #4 cite), recall that analytical means that the solution is an algebraic equation(s), whereas numerically means a table of numerical values. Both are plotted on a graph for comparison in order to decide if they're correct.



Mathematica also allows us to visualize their exact analytical solution in three dimensions. (see below) The three dimensional graph shows how the wave behaves for (0< t < 10). The z-axis (blue grid) represents the amplitude of the wave η_0. The black arrow represents the initial wave wave form, $f(y)$ at t =0.

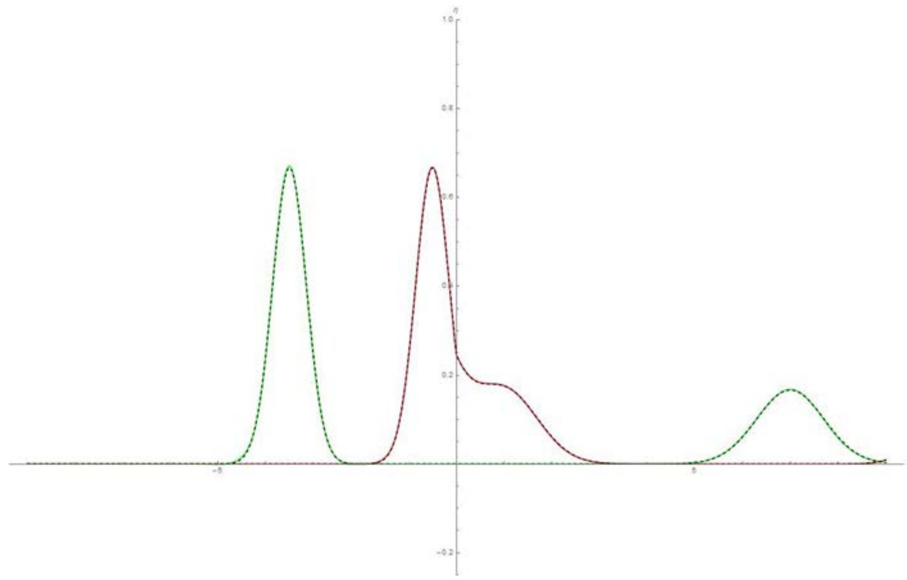


The other group's research investigated tsunami waves as they pass over shelfs and bumps rather than an infinitely sloped beach. In 2014 the REU used piecewise functions composed of line segments to model the ocean-floor topography, which would be continuous but not strictly differentiable at the "corners" where the linear segments connect (see figures below). In 2016 this group starts with the same equations (1, 2), and no moving boundary, then linearizes the equations into two sets of equations which are solved then solved using NDsolve in Mathematica. They approximate the vertical step shelves with a phi function that is smooth and continuous and uses this phi function, with the built in numerical analysis methods in Mathematica, to solve the problem both numerically and analytically.



They then systematized this process of curve-fitting with cubics to model any number of bumps of varying heights and widths. To check their results, they compare their results with the partial analytic solutions already developed [2].

Again, students used Mathematica to plot their solutions and compare the resulting graphs in order to validate their solution. The figure at right shows a linear wave as it moves over a shelf of height 0.75 at two different times. The analytic model, which should perfectly describe the wave moving over the shelf, is the black dashed line. Our numerical solutions are the colored lines, and they match up really nicely!



This was good because we were having difficulty making the numeric and analytic solutions line up with tall shelves (of *k* values greater than .9), but we just got this image over the weekend after messing with the step size and the size of the delta region to make our model fit even closer to a vertical step and finally got it without any discrepancies. Often it would take hours and hours for Mathematica to run one case and they would discover that there was a mistake or the computer would shut down or worse, display the blue screen sad face. These students bring multitasking to another level, often one student using up to three different laptops simultaneously.

Conclusions

Of all the REU's offered across the country, the REU at CSU Chico is one of only two (Illinois State University) that utilizes secondary math teachers. CSU, Chico provides in-service teachers with a research experience that will foster excitement about mathematics, increase content understanding, and inspire pedagogical innovation in their classrooms.

(Peter and Dan) Our conclusions are following:

We believe that the most accurate part of that statement is about excitement, When originally applying to this program, "The opportunity to do interesting mathematics with motivated intelligent people is irresistible." was included in both our applications. Between us, we have now participated in a total of six REUTs and this statement proved to be the most accurate description of our summer experiences.

As anyone who has talked with us since our participation will attest, we have not kept our enthusiasm for this program to ourselves. From having colleagues (math and non-math) cut up Mobius strips to trying to describe how you can model tsunami waves with Mathematica code we have spent hours talking to our colleagues about how much we enjoy this program. Through our roles as teacher leaders in our schools, our participation in the National Council of Teachers of Mathematics, the California Mathematics Council and the Mount Lassen Math Council, we have encouraged others to become a part of this program. Several of our friends and colleagues have applied for and been accepted to the program.

As teachers of Mathematics, we thoroughly enjoyed the problem solving process and have loved the opportunity to do challenging mathematics. Given the nature of the REUT, our personal growth as mathematicians has not been surprising. However, the depth of that growth has not only been surprising, but immensely satisfying. We have especially appreciated working on problems that have not been fully explored, as this is not the kind of mathematics we do as a regular part of our teaching. There is an adrenaline rush associated with taking the "road less traveled" or in this instance, going down a new road. The fact that the body of knowledge of mathematics is just a little bigger because of the mathematics we have done during our summers in the program continues to be a great source of pride.

Working with young mathematicians was also a major plus. We were able to use our years of experience as classroom teachers to help students develop their presentation skills and help with group dynamics issues. It has also been fun to watch as students helped us when it came to doing higher level mathematics that we had not seen in some time or in some cases never seen at all.

As we have stated many times in many contexts (four times in this paper alone) when describing this program, "The opportunity to do interesting mathematics with motivated intelligent people and be treated like a professional is irresistible."

As teachers, we have begun to recognize the importance of coding in the pursuit of mathematics. Whereas coding does not, and probably will not permeate our classrooms, programming is now a part of the higher levels of math we teach. Most, importantly we now recognize that coding will be a major part of any of our students' lives that decide to pursue mathematics as a career and it is of utmost importance that our students recognize this.

One of the biggest quandaries that we now have as classroom teachers is how do we bring the excitement that we have experienced when doing research into our classrooms? The math that we teach is complete. The algebra we teach is a finished curriculum. There is nothing that is not known. They know it, they know we know it. They know if they wait long enough we will tell them what they need to know. We know that learning is best done with a constructivist process, so the question is, "How do we change what our students do so they can experience the kind of excitement we have when doing research?" Does it have to be something that has never been done before or can it just be something that is just new to them? This was not a problem before we spent our summers doing research, but now we think about it whenever we are designing curriculum, reviewing curriculum or just planning a lesson.

(Peter) I teach in Alternative Education and would describe my students as "at-risk". This student population, in large part, lacks the will to persevere in the face of a problem. They are in the doldrums, unable to put the wind into their own sails and lack motivation. When I reflect on how this research experience has changed me. I return to the feeling I had when I first looked at the board filled entirely with abstract equations which at first glance, seemed incomprehensible. Doing mathematics gets challenging for everyone at some point in their math trajectory. If we reflect on mathematics as a language, the divergence between the undergraduates doing research and the students that I teach during the school year may best be illuminated within the construct of Krashin's Affective Hypothesis of Second-Language development.[11] The discrepant variables for these two groups of learners include motivation, self-confidence and anxiety. Krashen claims that learners with high motivation, self-confidence, a good self-image, and a low level of anxiety are better equipped for success in second language acquisition. Low motivation, low self-esteem, and debilitating anxiety can combine to 'raise' the affective filter and form a 'mental block' that prevents comprehensible input from being used for acquisition. I can honestly tell my students that it won't be easy, but it will be worth the effort… that their "I will" is more important than their IQ. I can now relate a shared experience which helps me build connections with my students, helping to lower their affective filters, a necessary step to facilitate their learning. Now I can truly empathize, fostering the relationships which can inspire them to persevere.

If you are part of a Research Experience for Undergraduates National Science Foundation funded program and you wish to increase the effectiveness of your program, the quality of your local teaching force and the inquisitive nature of your incoming freshman you might want to consider adding a T to your REU.

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