

Listening to Children Doing Math

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Abstract

Teacher preparation programs are likely to have perspective elementary teachers visit classrooms to observe mathematics lessons and help individual students with their work. The programs may also have courses in which common elementary student errors are explored. One professor has had perspective elementary teachers go into classrooms to ask individual elementary students mathematics questions and study each student's responses. The pre-service teachers observe each student's work, listen to each student's explanations, and monitor each student's body language. This opportunity can be beneficial for both parties. The perspective teachers learn about students' comprehension as well as their misunderstandings. The elementary students have the opportunity to practice communicating mathematically by explaining their work.

Listening to Children Doing Math

Introduction

In a New England university students who intend to become elementary school teachers must enroll in a course, Structures of Mathematics I: Number Systems. The course is taken by freshmen and sophomores and emphasizes problem solving, thinking and writing about mathematics, and looking for error patterns in elementary students' work. Most students in the course are surprised by, what is for them, a new approach to mathematics and are challenged by the need to reflect upon and explain their work and not simply employ algorithms to find answers to mathematics questions. One long-standing requirement of the course has been to observe mathematics lessons at two different elementary grade levels in two diverse communities. Recently, this professor has added an assignment for each pre-service teacher (PT) to interview an elementary student (ES). The interviews consist of asking the ES two mathematics questions, encouraging the student to explain the work, and observing the ES's attitudes towards the work. The primary intent of the assignment is twofold. First, it allows for the PTs to reflect upon the difference between a lesson, when it may appear that every ES understands the concept presented, and what each ES actually comprehends. Second, it introduces the fact that ESs may think about mathematics in ways different from the ways in which PTs think about it. The assignment is usually quite revealing to the PTs.

Literature Review

The traditional way to assess students' knowledge of mathematics is to utilize written work. That work frequently consists of tests which may be true-false, multiple choice, fill-in-the-blank, or may even require students to show or explain their work. However, all those assessments, as worthwhile as they are, may not disclose students' misconceptions or gaps in

their understanding. Furthermore, as Ginsburg (1997) and Heid (2007) explain, such written formative and summative assessments are not customized for individual students so they do not probe deeply into each student's knowledge. Heid (2007) and Greer (2009) point out that students may simply regurgitate correct words without truly understanding what they are writing, thus producing miscommunication or non-communication. Marilyn Burns (2010) points out that other assessment approaches such as one-on-one interviews are traditionally used by elementary teachers (ET) to assess students' reading comprehension. She and Small (2009) suggest that such approaches should be used for determining students' understanding of mathematics as well. Sometimes correct answers on written assessments may conceal a student's misconception that might only show up in a variation of the question asked or if an entirely different question about the same concept were asked. Burns also notes that oral questioning in the classroom in front of an entire class can be problematic. Too often it is utilized only if a student offers an incorrect answer. So, if a student is given a follow-up question after offering an answer, the student assumes that the original answer was incorrect.

Grandau, Landis, & Ryan (2007) suggest that it is important to assist PTs in making a connection between the theories to which they are introduced in their classrooms and the mathematical thinking which occurs in the elementary classroom. They point out that PTs should experience what mathematical ideas support ESs' work. In addition, PTs should learn good questioning skills so that in the future they will be able to uncover their own students' mathematical thinking. They and Burns agree that questioning students can inform instruction as well. Thus, when students are interviewed they might be told that the purpose of the interview is to help the interviewer become a better teacher. In other words, the more information teachers have about their students' understanding and misconceptions, the better prepared they will be to

be effective teachers. In addition, if the interviewer tells the ES that the interview is for the interviewer's sake, it can put the ES more at ease.

Burns (2010) and Ginsburg (1997) suggest that interviewing students provides an excellent avenue to connect with students on an individual basis. In other words, the interview process is not solely an exercise to determine understanding of mathematics. It is important to get to know one's students as individuals. Two specific instances where this idea applies are teaching ESL students and African American students. For work with ESL students, Lee, Silverman, & Montoya (2002) write that questions addressed to individual students should be short, clear and include an action. They suggest, for example, that asking an ESL student for an explanation of a rectangle is not as effective as giving the student manipulatives and asking the student to construct a rectangle.

Orr's (1987) and Sheppard's (2009) work with African American students emphasize how important it is for teachers, the majority of whom are not African American, to build a bridge between their own experiences and culture and their students' experiences and culture. Interviewing African American students can create a positive identification with them and can determine how they think mathematically. Both authors demonstrate that teachers should unearth the hidden and unmet potential, as Sheppard (2009) puts it, of their students. If teachers of African American students teach in a culturally responsive way they will validate the students' culture and life experiences as well as discover their understanding of mathematics. This idea can extend to most classrooms. If the classroom teacher is from a different socio-economic class and ethnicity than his/her students, then one-on-one interviews allow for the teacher to understand whence the students are coming. When teachers in those classrooms interview their students, they will not only determine their students' understanding of mathematics but will also

learn of their students' experiences and what is important to them. This information can both inform instruction and also provide ideas for contexts in which to present new mathematical ideas.

The Interview Assignment

The interview assignment requires each PT to ask an ES two different mathematics questions (see Appendix for sample questions). In most cases the PTs visit a school during one of their class meeting times. The public school which the PTs visit is a new school with an innovative college preparatory program including support from the New England university in which this assignment is given. One of the goals of this school is to inculcate the expectation of attending college, so the school encourages and welcomes visits from university students. As a consequence, this assignment results in a mutual benefit for both parties. If the course is offered in the summer or at a time during the academic year which does not coincide with the school's schedule, then the PTs find and interview an ES on their own.

In preparation for the interviews, the PTs are shown brief videos of three different ESs being asked mathematics questions. The professor distributes instructions with suggestions for carrying out the interview. The instructions encourage the PTs to:

1. spend a few minutes after sitting down with the ES to engage in conversation to get to know something about each other;
2. allow time for the ES to think about each question;
3. ask the ES how he/she arrived at the answer;
4. Only ask questions and not correct errors.

Once the interview is complete, the PTs are to prepare a written report on the experience, focusing on these questions.

1. Did the student speak up readily with descriptions of his/her work?
2. Did the student need much encouragement to explain his/her work?
3. Did the student know more or less than you expected?
4. Could the student describe his/her work clearly?
5. Did the student just do the work but not explain it?
6. Were the ES's explanations what you expected?
7. Would you have given similar or different explanations had you been asked to answer the questions?

In addition, the PTs are asked to compare and contrast their observation of a mathematics lesson with the one-on-one interview of an ES.

Finally, at the class meeting after the interviews take place, the PTs report orally on their various interview experiences. The oral reports provide the opportunity for the entire class to hear the great variety of the ESs' knowledge of and attitudes toward mathematics as well as the ESs' willingness and ability to explain their mathematical thinking. See Table 1 for PTs' comments after the interviews took place one semester.

Table 1

PT Comments about Interviews

PT Feelings	Multiple PT Observations	Single PT Observations
Nervousness entering the school where the interviews took place	During a mathematics lesson one can only tell what ESs who raise their hands think or know	ES needed encouragement to talk about what he/she was doing.
Fear that the ES would ask a question for which the PT could not provide an answer	By knowing what ESs think a teacher can draw on that knowledge to develop lessons	ES would look for interviewer's reactions before finishing work
More enjoyable than observing a mathematics lesson	The interview tests the patience of the PT. One must stay calm and be understanding	ES has more exciting way to explain mathematics than the PT has
Fun getting to know the ES	When asked to give an explanation, ESs said they didn't know	Work was sprawled all over the paper
	ES gave vague explanations	ES talked out loud while doing work
		ES revealed that he/she liked math until it came to long division
		ES drew pictures and explained work step by step

Limitations

One drawback of the assignment is that the PTs have only a single interview experience with one ES in one grade at one school. Time within one semester does not allow for additional interviews. Secondly, the PTs are most frequently freshmen and sophomores, so they have taken few, if any, education courses or may not have much experience working with ESs in an academic setting. Additionally, the PTs are enthusiastic about teaching and therefore have a difficult time simply asking mathematics questions and probing for knowledge. If an ES shows misunderstanding or cannot answer a question, the PTs want to clarify or teach the misunderstood concept. Another drawback is that good interviewing, as Ginsburg (1997) emphasizes, comes from extensive planning, practice and experience. Good interviewing is an art developed only over time.

Conclusion

The assignment regularly has very positive outcomes. The PTs are exposed to a different perspective on student understanding of mathematics which they had not previously considered. They begin to see that a) teaching a lesson in which students don't ask questions does not mean that all students comprehend the lesson's concepts and b) just because curricula state that students should learn specific concepts in each grade doesn't mean that they have. The PTs also begin to understand the relevance of topics of the course in which they are enrolled, such as numbers in different bases, multiple ways to carry out arithmetic and solve problems, and student errors. The wide variety of ES attitudes towards and abilities in mathematics becomes clearer to the PTs after the interviews. The ESs' struggles, not apparent in their observations of a mathematics lesson, become evident as well.

Appendix

Question 1.

You have been asked to make coffee cake for a school party, using a recipe that calls for the following ingredients:

2 cups sifted flour

2 eggs

1 cup sugar

1 cup sour cream

1 teaspoon vanilla

1 teaspoon baking soda

1 teaspoon ground cinnamon

$1\frac{1}{2}$ teaspoons baking powder

$\frac{1}{2}$ cup butter

$\frac{1}{4}$ teaspoon salt

The recipe you have was intended to feed five persons, but you need to prepare enough for all 60 students in your grade.

- How would you adjust the recipe to feed 60 students?
- List all of the ingredient amounts needed for the adjusted recipe.

Taken from *Middle School Mathematics I*, Springboard Mathematics with Meaning (2005).

Question 2.

Long ago, people observed the sun rising and setting over and over at about equal intervals. They decided to use the amount of time between two sunrises as the length of a day. They divided the day into 24 hours. Use what you know about numbers to answer these two questions.

- a. Why is 24 hours a more convenient choice than 23 or 25 hours?
- b. If you were to select a number different from 24 to represent the hours in a day, what number would you choose? Why?

Taken from *Illuminations*. National Council of Teachers of Mathematics (2008)

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