Using Electronic Reflections to Help Pre-Service Elementary Teachers

Learn to Communicate Mathematically

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Abstract

Having undergraduate pre-service elementary teachers write electronic reflections as part of their mathematics courses has multiple benefits. Both students and instructors gain insights from the process. Writing reflections gives students time to think critically about concepts outside of class and see mathematics as more than calculations and algorithms. It provides opportunities for students to communicate mathematics effectively. Students can utilize reflections for reviewing for tests. Instructors can gain insight into students' thought processes and comprehension and can respond to students in a timely way. This paper enumerates the hurdles and multiple benefits of having students write electronic reflections.

One of the National Council of Teachers of Mathematics' goals for successful mathematics education is for students to gain the ability to communicate mathematically. One of the ways in which communication can take place is through the written word. Bradenburg (2002) and Isom (1996) say that writing is an excellent tool for extending and deepening students' understanding and is worth the effort it takes to introduce it into the mathematics curriculum. Burns (2004) says that writing can help students become more precise in their mathematical thinking and allow them to reflect upon mathematical ideas. As Hershkowitz (1999) put it, writing allows for a purification process. It requires students to put various mathematical ideas into words and consider the process as well as the product. Similarly, Ediger (2006) proposed that students "learn to write as well as write to learn mathematics" (p. 120). He suggested that to write in mathematics students must analyze, synthesize, and evaluate their experiences.

Borasi and Rose (1989) suggested that writing in mathematics can assist students to move beyond thinking that mathematics is simply acquiring or memorizing facts and algorithms which together can be utilized in answering questions in textbooks. Writing, they advocated, can lead students to perceive of mathematics as a meaningful and creative discipline. Although students perceive of a great gulf between mathematics and writing, Borasi and Rose recommended that "writing to learn can provide a valuable means to facilitate a personalized and making-of-meaning approach to learning mathematics" (p. 347).

Almost forty years ago an educational reform movement, referred to as Writing Across the Curriculum (WAC), grew as a result of some educators' belief that education should allow for creativity and not depend upon rote learning and lecture. Research conducted over the years since the birth of WAC suggests that WAC allows for more creative and active learning (University of Missouri St. Louis, 1996). Recently, various mathematics educators have investigated writing in mathematics at various levels of education. In South Africa, Ntzena (2006) investigated seventh grade students who were engaged in a variety of writing-to-learn activities including written assignments, investigations, and journals. He found that a) writing is worthwhile if it is regular and frequent and b) teachers must become informed about how to implement writing in the mathematics classroom. Brandenburg's (2002) thinking parallels Borasi and Rose's. She emphasized that mathematics students in upper-level high school courses were able to complete exercises but were unable to comprehend the relationship of concepts or to use their knowledge in unfamiliar territory. She used writing in mathematics to help students think at higher levels in mathematics. Brandenburg found the task of getting her students to write an arduous but very worthwhile task. The end results were an enormous increase in comprehension and an ability to explain their knowledge. Community college students who wrote a book on fractions gained a sense of empowerment as they explained mathematics and took ownership of the ideas they wrote about (MacLeod, 1992). They also learned to ask good questions.

Writing in the Mathematics Classroom

All students write in the mathematics classroom, but their writing is frequently limited to what Britton (1975) would call transactional writing. That is the recording of facts, the explanation of ideas, the construction of theory, the copying or transcribing of an instructor's notes or classmates' work, and the summarizing or interpreting of other people's ideas. It is likely to be direct copying of information, translating mathematical symbols into words, summarizing, and answering test questions. It is less likely to be what Britton would call expressive writing, where students are thinking aloud on paper or electronically.

Traditionally, students are asked to solve word problems where they need to translate facts already in sentences into equations. Students are less likely to be asked to translate facts already in equations into word problems. Professors may not ask students to think or reflect about mathematics at all or only infrequently. Student questioning beyond “Am I doing it right?” or “Is this the right answer?” may not happen much in the classroom. In fact, teachers may not be certain about how to encourage students to utilize expressive writing or ask thoughtful, meaningful questions in mathematics. If teachers have not had the experience of writing in mathematics courses in which they have been enrolled, they may not be able to model expressive writing in their own classrooms.

A weakness of the American public school system is that it does not inculcate the practice of reflection (Senge, 1990). Schőn (1983) and Senge (1990) suggested that the learning process requires reflection within practice. Swan (2000) and Jaberg, Lubinski & Yazujian (2002) determined that students should be given time to struggle and reflect upon mathematical ideas. One form of reflection can be expressive writing. Reflecting, in the form of writing, takes time and thought. It requires writers to ponder their ideas about mathematical concepts, formulate their thinking, and express their thoughts in a clear, concise way. It may take considerably more time than completing a standard homework assignment of executing exercises and solving word problems. However, reflection in the form of writing can assist students to find meaning in mathematics and even ponder extensions of ideas presented in class. Recently published college mathematics textbooks (Bassarear, 2007; Bittinger & Ellenbogen, 2008; and Stewart, Redlin & Watson, 2006; Young, 2009) include exercises which ask reflective questions, so it appears that some textbook authors recognize that writing in mathematics is a worthwhile endeavor.

While it has been demonstrated that writing in the mathematics classroom helps students gain deeper understanding of mathematics, expressive writing in a mathematics class is a relatively new idea. University faculties should consider how they might prepare pre-service elementary teachers to reflect and write within mathematics. If pre-service teachers experience writing in their own university mathematics courses, then they might see reflective writing as a normal expectation in mathematics instruction and thus incorporate it in their own classrooms some day.

Electronic Reflections Assignment

At a state university in New England several courses are offered for pre-service elementary teachers who are majoring in mathematics. Students traditionally enroll in two of the courses, Number Patterns and Development of Geometric Ideas, during their junior year. The professors who teach these two courses require that their students submit electronic reflections on a regular basis at least ten times during the semester.

The assignment has three components which ask the questions:

- What are the big ideas you learned during the week? Please state the ideas with some explanation and examples in your own words.
- Were there any new vocabulary words or phrases with which you became familiar? Write a brief definition of each in your own words.
- What are some questions you still have or ideas you would like to pursue further?

This assignment has manifold purposes and intends to assist both the students and the professors. The students have the opportunity to:

- reflect upon the ideas and concepts to which they have just been exposed,
- practice explaining mathematics,
- write about mathematics,
- use suitable mathematics vocabulary in appropriate ways,

- receive immediate clarifications and corrections if they have any obvious misconceptions, and
- ask the professor questions which were not posed during class.

The professors can:

- find out what the class as a whole understands,
- determine what misconceptions exist,
- respond to individual students about isolated misconceptions, and
- incorporate reviews of ideas and concepts in the next lesson if a majority of the class needs clarification of any of the ideas or concepts.

Outcomes

An assignment to write about mathematical ideas is new to most of the students enrolled in these courses. They may have had a brief exposure to writing in earlier collegiate mathematics courses when they were assigned exercises from their texts which asked for explanations of how they carried out a particular calculation. They may not have had experiences of describing ideas or concepts in their own words or of thinking beyond what had been presented in class. At this university, however, students are exposed to similar writing assignments in several mathematics courses. The first time they encounter the reflection assignment they see it as a new idea, but by the second course in which they encounter the assignment they are not surprised by the expectation.

At the beginning of the first of the two courses, students' early reflections tend to be very brief and consist more of transactional writing than expressive writing. They are more likely to provide lists of topics covered in class or examples without explanations. Many times the reflections are vague and are less likely to provide both the explanations and the examples which the assignment requires. Students may copy explanations of concepts instead of writing the explanations in their own words, and they are also likely to present the professors' examples. Sometimes their explanations are faulty. What follows are examples of the first and seventh reflections provided by a few of the students in Number Patterns. These examples are included without edits, exactly as the students wrote them. Improvements in students' reflections vary a great deal, but frequently students show some growth in giving their own explanations and examples of topics with more clarity even if they do not reach the assignment's expectations or they have some errors.

Student A (traditional-aged male)

Reflection 1

The number 36 is:

I. Even
II. Perfect square
III. Divisible by: 1,2,3,4,6,9,12,18, and 36
IV. Abundant
V. Composite

Expressions:

I. (n(n-1)/2; a way to find how many handshakes there would be if n number of people all shook hands
II. (n-k)+1; a way to find the number of whole numbers between k (the smaller number) and n (the larger number)

I can use this expression to find:

I. How many even or odd numbers are between two numbers
II. How many numbers are between two numbers that are divisible by any integer

Reflection 7

I found the bases are particularly interesting. A good example of the bases is base two. The first decimal place can hold zero to one and is the ones place. The second decimal place can hold zero to one and can hold up to two numbers. The third decimal place can hold zero to one and can hold up to four numbers. The decimal places follow this pattern of two to the zero, two to the first, two to the second, and so on.

Student B (non-traditional-aged female)

Reflection 1

> The many concepts of counting numbers using mathematical observations and interpretations has always intrigued me. This week my mind was reawakened to mathematical logic and terminology I have not heard or come across in some time, if I have, I have not taken much notice as I have this week. One mathematization I have encountered this week is the simple formula to solve the number of whole numbers between k and n inclusive is n-k+1. During classwork we applied an algebraic equation to prove a mathematical theory: that when you toss a standard pair of dice once, following a certain sequence of applied mathematical functions relating to the dice, the sum of the results (pertaining to the sequence of functions) will always equal 49.

Reflection 7

> One idea we learned this week in class is about the different bases of numbers. I know our number system is based on base 10, meaning our digits include 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. I learned when we use different bases the digits will begin at 0 and the last digit used in a certain base will be n-1 (with n representing the base number being used.) Example: base 7 will include the digits 0, 1, 2, 3, 4, 5, and 6. The number of each space, (meaning the number represented by a certain base number) is determined by the powers of the base. The far most right digit represents (how many being used) the base number to the zero power, the second right most digit represents the base number to the first power, the third right most digit represents the base number to the third power, etc.

Student C (traditional-aged male)

Reflection 1

> The big idea this week was patterns, which is what this course is all about. The first few assignments really drove this point home by concentrating on problems which could be solved by finding patterns.

Reflection 7

> This week there were two big ideas that we looked at. The first was the binomial theorem. This is a theorem that deals with expanding a sum exponentially. For example, if we had to expand $(x+y)^6$ we would be able to do this quickly using the binomial theorem. Methods that we used included using Pascal's Triangle to find the coefficients, the sum of each of the terms exponents are always equal to the power the sum is being raised to, and the powers decrease for the x term and increase for the y term. So in order to do this example I would have to refer the sixth row in Pascal's Triangle in order to get the coefficients, and I know that the first x term would be raised to the sixth power and the first y term would be raised to the power of zero with a coefficient of one to yield x^6. From here I would follow the pattern we learned to come out with the term $6x^5y^1$ as next.

Student D (traditional-aged female)

Reflection 1

An expression is when a variable such as “n” is manipulated to represent how the initial amount of what you were given changed to determine what you will end up with. For example: let’s say you were given a perfectly square peice of land and wanted to know how many sq. feet you’d be able to gain by expanding a foot in each direction. You could use an expression with “n” representing the amount of land you want to add. If your initial amount was 90,000 sq. feet of land then the expression to determine the amount of sq. feet you gained by adding a foot in each direction would be $90000\text{-}((300+n)^2$. The second topic we covered was using multiplication to count. A situation in which you may need multiplication to count would be if you were at Dunkin Donuts and wanted to know how many different breakfasts you could have. If there were 6 different hot beverages you could choose from, 3 different breakfast sandwiches, and 9 kinds of donuts, you would multiply 6 by 3 by 9 to determine how many different ways you could choose. If you to decide between a breakfast sandwich or a donut you would multiply 6 by 3 and then add it to 6 by 9.

Reflection 7

Euclid’s Algorithm is a method of finding the greatest common factor of two numbers. The greatest common factor of two numbers is the largest number that divides both numbers evenly. In Euclid’s Algorithm you derive the greatest common factor by divided the smaller number (b) into the larger number (a) if there is no remainder then the smaller number (b) is your greatest common factor. However if there is a remainder then you now must call the remainder (b) and your previous (b) is now the (a) because it is larger than the remainder. You now, once again divide (b) into (a), if there is no remainder then the greatest common factor is (b), but if there is a remainder then you once again rename (a) and (b) and repeat the steps until there is no longer a remainder. An example of find the GCF (greatest common factor) using Euclid’s Algorithm is given 81 and 45 you would divide 81 by 45 and get 1 with a remainder of 36. Nests you would divide 45 by 36 and get a remainder of 9. Then you would divide 36 by 9 and get 4, with no remainder. Since 9 divides 36 evenly 9 is your greatest common factor.

Student E (traditional female)

Reflection 1

During the week I learned that an explanation of why certain things happen with certain numbers are important. It is important to find the solution to a question, but it is even more important to be able to tell how that answer came about and figure out if there are any other ways to get the same answer. Adding, this week we learned about abundant, deficient, and perfect numbers. I learned why the number 36 is a special number and one reason is that the number 36 is abundant.

Reflection 7

> While Student E's seventh reflection continued to fall short of the assignment's expectations, she did offer more detail than she did in her first submission and wrote expressions correctly. "During last week's classes the big ideas were sequences and collections. The difference between a sequence and a collection is that in a sequence order matters and in a collection order doesn't matter. We learned how to solve a variety of sequence and collection problems and learned the equations for both. For example, when order matters we can use the formula: n!/((n-k)!) and when order doesn't matter we use the formula: n!/((n-k)!(k!)).

The second component of the assignment asks for definitions of new vocabulary. When it comes to providing definitions of words or phrases in their reflections, frequently students adopt the same approach which they use for the first component of the assignment and simply provide a definition from their notes. In other situations students may say, without actually writing definitions, that they are already familiar with the vocabulary introduced during lessons because they previously encountered it. In these cases it is not possible to know whether the students could actually define the vocabulary accurately. In many cases students simply omit addressing the vocabulary component of the reflection assignment.

In response to the third component of the assignment, the majority of students write much less than they write for the first two components, frequently stating that they have no questions. When they do pose questions, the questions are usually asking for clarification of ideas presented in class. Their questions are less likely to address extensions of ideas already presented in class or to pursue additional related ideas, at least in the first course in which they are required to write reflections. They may, however, ask how they might use the concepts presented in class. Their emphasis at this point is usually on concrete examples much like the old adage "when are we ever going to use this?"

Since some students really struggle with this reflection assignment when they first encounter it, it is advantageous to supply samples of other student reflections from previous semesters. Samples of what the professors deem excellent, good, and inadequate reflections, along with justifications for each sample's rating , are offered at the beginning of each semester. Additionally, when thoughtful comments or questions submitted by students during the semester are shared with the entire class, it provides an impetus for other students to attempt the task in subsequent reflections. In general, as the semester progresses and with frequent feedback from the professors, the responses improve.

After the first course in which electronic reflections are assigned, students are likely to take such an assignment for granted when they next meet it and to be more successful in their writing. Hopefully, with the multi-course experiences of writing reflections in mathematics, the pre-service elementary teachers will come to appreciate the value of writing in mathematics and will incorporate writing in mathematics in their own classrooms some day.

Observations

Not all students complete the assignment and submit the ten reflections. However, the students' reflection submission rates are comparable to their traditional homework submission rates. In Number Patterns the grading of the reflections and the homework is the same, and the weight assigned to both is equal. The reflections are graded in a holistic manner as excellent, good, or inadequate. Excellent reflections are ones which address all three components of the assignment, a) carefully explaining one or more concepts with the student's own example(s),
b) carefully explaining one or more vocabulary with the student's own example(s), and
c) mentioning an extension of the concept(s) presented in class either in the form of asking a question or in the form of referring to an internet search on the concept. They receive a grade of 110%. Good reflections are ones which fall short of excellent ones because one of the three components is missing or very weak or there are significant errors in the explanations. They

receive a grade of 90%. Inadequate reflections are ones where two of the three components are missing or are very weak or show serious misconceptions. They receive a grade of 70%.

Perhaps students perceive of writing reflections as just another homework assignment in addition to completing a homework set assigned from a text or on a handout. Writing weekly reflections may initially go beyond the students' comfort level. Most students begin the fourteen-week semester enthusiastically submitting their first reflections at the end of the first week. There are some students, however, who procrastinate for several weeks, not submitting their first reflections until there are just enough, or not enough, weeks left in the semester for submitting all ten reflections.

Typically, female students complete the assignment more faithfully than male students. Additionally, most men's reflections are considerably shorter and more likely to be outlines of topics covered with little explanation and no examples. It is not clear whether the men do not see the importance of the assignment, are uncomfortable with writing in general, or have more difficulty in writing about mathematics. One very able male student who participated in class and was enthusiastic about helping classmates who were struggling in the course had difficulty with the writing assignment. He admitted to the professor that he had difficulty getting motivated to complete the writing assignment.

Over a period of five consecutive semesters 85 students were enrolled in the course, 78 of whom were female and seven of whom were male. This preponderance of women to men is not uncommon since fewer men seek to become elementary school teachers. Table 1 illustrates the students' reflection submission rates by gender. While there were three women who submitted fewer reflections than any man, 94% of the women submitted at least eight while only 57% of the men submitted at least eight.

Table 1

Frequency of Reflection Submissions

Number of Student Submissions	11	10	9	8	7	6	5	4	3	2	1	0
Females (n=78)	6	52	11	4	1	1	0	2	1	0	0	0
Males (n=7)	1	2	1	0	2	0	1	0	0	0	0	0
Total (n=85)	7	54	12	4	3	1	1	2	1	0	0	0

Table 2 illustrates, by gender, students' average number of submission, average grades for the reflections they submitted, and their average reflection grade for the semester based upon ten reflections. While the male students had good average grades for the reflections which they did submit, their reflection grades for the semester suffered, because on average, they submitted fewer than the required number of reflections.

Table 2

Average Grades for Reflection Submissions

Number of Student s	Average Number of Submissions	Average Grade for Submissions	Average Grade for Semester
Females (n=78)	9.5	102%	97%
Males (n=7)	8.4	100%	83%
Total (n=85)	9.4	102%	96%

Additional Student Input

The end results of this assignment go far beyond the anticipated outcomes. In addition to addressing the three components of the assignment, students may voice their frustrations or confusions. They may also complain. Complaints may be about any number of things such as the text, other students, or tests. Students may write about situations with which they are uncomfortable or state concerns which they choose not to verbalize during class. They may write about a concern which they may have wanted to ask in class had there been enough time.

Students may also write comments they would be uncomfortable making face-to-face to the professor or the class. During one semester there were a handful of very vocal students in one section who frequently complained about a variety of things. Interestingly, there were just as many students in the same section who wrote in their reflections that they did not agree with their vocal classmates. It appeared that the students who commented in their reflections about their disagreements with the vocal students were intimidated by their more vocal classmates but utilized the electronic communication format to express their own opinions and feelings privately.

Other input from this assignment includes the occasions when students report on their surprises or "aha" moments in the course or describe connections that they see for the first time. One student reported: "what I am happy about is that I get the proof. I finally get it its not just memorized I get it [*sic*]."

Many students obviously reviewed their notes taken in class before writing their reflections. Some stated that in reviewing their notes they realized that they were unclear about some concept or procedure. Some students commented on relationships they observed between the content of the course and some other mathematics course they had or were taking.

Requiring students to submit reflections *via* email fosters additional student/professor communication. This more open-ended task, which asks for the pursuit of additional ideas, encourages students to take risks with higher level thinking. In addition, it opens the door for students to report to their professor some ideas and feelings that they might not otherwise take the time to stop and share with the professor. After a discussion in one class about whether zero is an even number or not, one student reported electronically that she had been taught that zero is neither and that "this idea was strictly enforced during calculus that I took senior year of high school." This student asked in her reflection if she should bring that idea to class or just "keep in mind that zero is in fact an even number." Such a student may not have risked raising that concern during class, but she felt comfortable raising it in her reflection. Once the professor validated her concern in a response email, the student may have felt safe mentioning her other ideas in class.

Students are encouraged to use correct mathematics vocabulary, proper grammar, and correct spelling when writing their reflections. As pre-service professionals, this is an important venue for the students to learn to communicate accurately in their own lessons, with their colleagues, and with their students' parents.

In one section of Number Patterns, when it came time for the first test, the electronic communication was extended. Since the reflections were seen as beneficial for student-instructor communication, students were asked to prepare questions to review for the test and submit them to the professor electronically. This task required the students to begin their review for the test well before the time of the test, and it had the students, rather than the professor, direct the review. Once the professor received the questions, she created a practice test based on the students' questions and sent it to the students electronically so that they could complete the review before the class meeting immediately prior to the test. The students received this assignment enthusiastically and asked for the same procedure prior to the remaining tests and final exam.

Summary

Numerous researchers and theorists, along with NCTM, agree that writing in and about mathematics is an essential component to gaining a better and deeper understanding of mathematical concepts. In the preparation of pre-service elementary teachers one university requires students to write frequent reflections on a regular basis in multiple higher level mathematics courses. Initially the task is difficult for many students, primarily because they are not accustomed to writing in or about mathematics. As the first semester in which this requirement is assigned progresses, the students move from listing ideas and supplying examples of topics to providing some more in-depth explanations. Students make even more progress in writing clear, thoughtful reflections in subsequent courses. Student-professor communication is fostered. Students include in their reflections comments or questions they do not have a chance to raise in or unable or uncomfortable to ask in class, and they get immediate feedback from their professors. Professors discover and are able to clarify misconceptions immediately, and a venue for students to take risks with higher level thinking is established. Hopefully, the pre-service elementary teachers gain an appreciation for writing in mathematics and will include it in their own classrooms some day.

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